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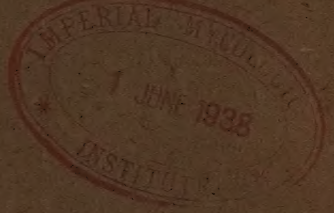

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MATHÉMATIQUES

SUR L'APPLICATION DE L'INTÉGRALE DE FOURIER AUX ÉQUATIONS INTÉGRALES NON-LINÉAIRES

Par N. SMIRNOFF

(Présenté par S. N. Bernstein, de l'Académie, le 17. II. 1938)

L'intégrale de Fourier (ainsi que ses modifications) a été appliquée plusieurs fois à la résolution des équations intégrales linéaires. Dans la suite je propose, à ce qu'il me semble pour la première fois, quelques applications de l'intégrale de Fourier aux équations intégrales non-linéaires.

Lemme. *L'équation*

$$\xi^n + a_{n-1}\xi^{n-1} + \ldots + a_1\xi + a_0 = 0, \tag{1}$$

où

$$|a_i| < \frac{cA^n}{|s^b|^n} \qquad (i = 0, 1, \ldots, n-1), \tag{2}$$

la partie réelle $Rs = \alpha > 0$, $0 < A < \alpha^b$, $b > 1$ et

$$c = \frac{1}{2^n}, \tag{3}$$

a des racines en module inférieurs à $\frac{A}{|s^b|}$.

Démonstration. Désignons par $f(\xi)$ le premier membre de (1). Pour démontrer le lemme, il suffit de montrer que pour $|\xi| \geqslant \frac{A}{|s^b|}$ on a $|f(\xi)| > 0$. En effet, en vertu de (2) on a:

$$|f(\xi)| \geqslant |\xi|^n - \sum_{k=0}^{n-1} |a_k| \cdot |\xi|^k > |\xi|^n - \frac{cA^n}{|s^b|^n} \cdot \sum_{k=0}^{n-1} |\xi|^k. \tag{3'}$$

Pour $|\xi| \leqslant 1$ on aura:

$$|f(\xi)| > |\xi|^n - \frac{cA^n \cdot n}{|s^b|^n},$$

ce qui est > 0 pour $|\xi| \geqslant \frac{A}{|s^b|}$, parce que $c \cdot n = \frac{n}{2^n} \leqslant 1$.

Pour $|\xi| > 1$ on a

$$|f(\xi)| > |\xi|^n - \frac{cA^n}{|s^b|^n} \cdot \frac{1 - |\xi|^n}{1 - |\xi|},$$

ce qui est > 0 pour $1 + \frac{cA^n}{|s^b|^n} < |\xi| < 2$, parce que $\frac{cA^n}{|s^b|^n} < 1$.

Ainsi on voit que toutes les racines de (1) $|\xi| < 2$. En posant dans le second membre de l'inégalité (3') $|\xi| = 2$, on a:

$$|f(\xi)| > |\xi|^n - \frac{cA^n}{|s^b|^n} \sum_{k=0}^{n-1} 2^k = |\xi|^n - \frac{cA^n}{|s^b|^n}(2^n - 1).$$

Cette dernière expression est > 0 pour $|\xi| \geqslant \frac{A|}{|s^b|}$, parce que $c(2^n - 1) < 1$.

Théorème 1. *L'équation intégrale*

$$K_0(x) = \sum_{k=1}^{n} \int_0^\infty \dots \int_0^\infty K_k(xy_1 \dots y_k)\, u(y_1) \dots u(y_k)\, dy_1 \dots dy_k \equiv \mathfrak{N}(x; y), \qquad (4)$$

où les K_k $(k = 0, 1, \dots, n)$ sont des fonctions partiellement unies (stück weise-glatte) et telles que les intégrales:

$$[K_k] \equiv \int_0^\infty x^{-s} K_k(x)\, dx \qquad (Rs = \alpha > 0,\ k = 0, 1, \dots, n),$$

vérifient les équations:

$$\left|\frac{[K_k]}{[K_n]}\right| < \frac{cA^n}{|s^b|^n} \qquad (k = 0, 1, \dots, n-1), \qquad (5)$$

$0 < A < \alpha^b$; $c = \frac{1}{2^n}$; $b > 1$, *possède au plus n solutions bornées et elles peuvent être exprimées dans la forme:*

$$u_r(y) = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} y^{-s} F_r(s)\, ds \qquad (r = 1, 2, \dots, n), \qquad (6)$$

où les $F_r(s)$ sont les racines de l'équation algébrique:

$$\sum_{k=1}^{n} [K_k]\, \xi^k - [K_0] = 0. \qquad (7)$$

Démonstration. En vertu de (5) l'équation (7) peut être divisée par $[K_n]$ et écrite dans la forme:

$$\xi^n + \sum_{k=1}^{n-1} \frac{[K_k]}{[K_n]} \xi^k - \frac{[K_0]}{[K_n]} = 0.$$

En vertu du lemme précédent on a pour ses racines $|\xi| \leqslant \frac{A}{|s|^b}$. Posons $\xi = \int_0^\infty x^{s-1} u^*(x)\, dx$; alors (7) pourra être écrite dans la forme:

$$\sum_{k=1}^{n} [K_k] \left(\int_0^\infty x^{s-1} u^*(x)\, dx \right)^k - [K_0] = 0$$

ou bien

$$\sum_{k=1}^{n} \int_0^\infty K_k(\omega)\, \omega^{-s}\, d\omega \int_0^\infty \dots \int_0^\infty (y_1 \dots y_k)^{s-1} u^*(y_1) \dots u^*(y_k)\, dy_1 \dots dy_k -$$

$$- \int_0^\infty K_0(x)\, x^{-s}\, dx = 0.$$

Cette dernière expression peut être écrite dans la forme:

$$\sum_{k=1}^{n} \int_0^\infty \dots \int_0^\infty K_k(\omega)\,\omega^{-s}\,d\omega\,(y_1 \dots y_k)^{s-1}\,u^*(y_1) \dots u^*(y_k)\Big]dy_1 \dots dy_k -$$

$$- \int_0^\infty K_0(x)\,x^{-s}\,dx = 0.$$

En faisant dans les intégrales $\int_0^\infty K_k(\omega)\,\omega^{-s}\,d\omega$, $(k = 1, 2, \dots, n)$, changement de variables en posant $\omega = xy_1 \dots y_k$, on a:

$$\sum_{k=1}^{n} \int_0^\infty \dots \int_0^\infty K_k(xy_1 \dots y_k)\,x^{-s}\,dx u^*(y_1) \dots u^*(y_k)\,dy_1 \dots dy_k -$$

$$- \int_0^\infty K_0(x)\,x^{-s}\,dx = 0. \qquad (8)$$

On peut dans ces intégrales changer l'ordre de l'intégration par rapport à x et à $y_1, \dots, y_k$, car les expressions sous le signe d'intégrale sont uniformément bornées et tendent uniformément vers zéro quand $x, y_1, \dots, y_k$ tendent vers l'infini, pourvu que a et $b > 1$ soient convenablement choisis; en effet

$$|u^*(x)| = \left| \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} x^{-s} \xi(s)\,ds \right| < \frac{A}{2\pi} x^{-a} \int_{-\infty}^{+\infty} \frac{dy}{|a+iy|^b}.$$

Donc, on peut écrire (8) dans la forme:

$$\int_0^\infty x^{-s} \left(\sum_{k=1}^{n} \int_0^\infty \dots \int_0^\infty K_k(xy_1 \dots y_k)\,u^*(y_1) \dots u^*(y_k)\,dy_1 \dots dy_k - K_0(x) \right) dx = 0.$$

En appliquant maintenant la formule de Mellin, on a:

$$\mathfrak{N}(x; u) - K_0(x) = 0,$$

pour toutes les valeurs de x, donc on obtient l'équation (4). Donc les $u^*(x)$ que nous avons pris sont des solutions de l'équation (4), ce qui prouve que $u(x) = u^*(x)$. Leur nombre est évidemment égal à n (mais elles peuvent ne pas être toutes différentes).

Exemples. L'équation

$$K_0(x) = \int_0^\infty K_1(xy)\,u(y)\,dy + \int_0^\infty \int_0^\infty K_2(xy_1y_2)\,u(y_1)\,u(y_2)\,dy_1dy_2,$$

où K_0, K_1 et K_2 vérifient les restrictions du théorème 1, possède deux solutions que l'on peut présenter dans la forme:

$$u(x) = \frac{1}{4\pi i} \int_{a-i\infty}^{a+i\infty} x^{-s} \frac{1}{[K_2]} \left(-[K_1] \pm ([K_1]^2 - 4[K_0][K_2])^{1/2} \right) ds.$$

L'équation

$$K_0(x) = \int_0^\infty \dots \int_0^\infty K(xy_1 \dots y_k)\,u(y_1) \dots u(y_k)\,dy_1 \dots dy_k$$

a solutions différentes exprimées par la formule:

$$u(x)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} x^{-s}\left(\frac{[K_0]}{[K]}\right)^{\frac{1}{k}} ds.$$

Théorème 2. *L'équation intégrale*

$$u(x)=\mathfrak{N}(x;u)+K_0(x) \tag{9}$$

pourvu que toutes les fonctions $K_i(i=0, 1, \dots, n)$ *soient partiellement unies et vérifient les équations*

$$|[K_n]^{-n}\{K_n\}^{-1}|<\frac{cA^{n^2}}{|s^b|^{n^2}}, \qquad \{K_k\}\equiv\int_0^\infty x^{s-1}K_k(x)\,dx$$

$$\left|\frac{\{K_k\}[K_{l_0}]^{\alpha_0}[K_{l_1}]^{\alpha_1}\dots[K_{l_k}]^{\alpha_k}}{[K_n]^n\{K_n\}}\right|<\frac{cA^{n^2}}{|s^b|^{n^2}}, \tag{10}$$

où

$$0\leqslant l_i\leqslant n;\quad \alpha_0+\alpha_1+\dots+\alpha_k=k;\quad 0\leqslant\alpha_i\leqslant n-1\quad (i=1, 2, \dots, n),$$

$$c=\frac{1}{2^n(n+1)^n}, \tag{11}$$

possède au plus n^2 *solutions bornées qui s'expriment par la formule*

$$u_r(x)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} x^{-s}\xi_r(s)\,ds \qquad (r\leqslant n^2), \tag{12}$$

où les $\xi_r(s)$ *sont les racines du système d'équations:*

$$\xi=\sum_{k=0}^{n}[K_k]\eta^k; \qquad \eta=\sum_{k=0}^{n}\{K_k\}\xi^k. \tag{13}$$

Démonstration. En multipliant (9) par $x^{-s}dx$ et intégrant dans $(0, \infty)$ on obtient

$$[u]=\int_0^\infty x^{-s}\mathfrak{N}(x;u)\,dx+[K_0]. \tag{14}$$

Ainsi qu'il a été fait précédemment on réduit cette équation à la forme:

$$[u]=\sum_{k=1}^{n}[K_k]\{u\}^k+[K_0]; \tag{15}$$

la légitimité de ce passage est démontrée dans la suite.

En remplaçant dans (15) s par $1-s$, on obtient:

$$\{u\}=\sum_{k=1}^{n}\{K_k\}[u]^k+\{K_0\}. \tag{16}$$

Les équations (15) et (16) présentent un système (13) avec les inconnues $\xi(s)=\{u\}$ et $\eta(s)=[u]$. En définissant $\xi(s)$ de ce système qu'il est facile de réduire à une seule équation en substituant le second membre de (15) au lieu de $[u]$ dans l'équation (16), ce qui donne:

$$\xi=\sum_{k=1}^{n}\{K_k\}\sum_{l=1}^{n}[K_l]\xi^l+[K_0]^k+\{K_0\}, \tag{17}$$

nous obtenons immédiatement $u(x)$ sous la forme (12) en appliquant la formule de Mellin à

$$\xi(s) = \int_0^\infty x^{s-1} u(x)\, dx.$$

Comme le coefficient de ξ^k $(k = 0, 1, \ldots, n^2 - 1)$ dans (17) est une somme de termes de la forme:

$$\{K_k\}[K_{l_0}]^{\alpha_0}[K_{l_1}]^{\alpha_1} \ldots [K_{l_k}]^{\alpha_k}$$

où $0 \leqslant l_i < n$; $\alpha_0 + \alpha_1 + \ldots + \alpha_k = k$; $0 \leqslant \alpha_i \leqslant n - 1$; $(i = 1, 2, \ldots, n)$, et leur nombre est $< (n+1)^n$, on peut appliquer aux racines de (17) le lemme en tenant compte des conditions (10) et (11) du théorème; cela nous donne:

$$|\xi(s)| < \frac{A}{|s^b|}.$$

Cela posé, en choisissant convenablement $\alpha > 0$ et $b > 1$, nous pouvons passer de (14) à (15). On obtient alors immédiatement l'existence des intégrales (12). Comme le système d'équations (15) et (16) a au plus n^2 solutions, l'équation (9) dans les conditions (10) et (11) possède au plus n^2 solutions. L'expression (12) est une solution de (9) dans le cas où $\xi(s)$ est une racine du système (12).

Exemple. L'équation

$$u(x) = \int_0^\infty \ldots \int_0^\infty K(xy_1 \ldots y_n)\, u(y_1) \ldots u(y_n)\, dy_1 \ldots dy_n$$

possède $n^2 - 1$ solutions qui diffèrent de zéro et qui sont données par la formule:

$$u(x) = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} x^{-s} \left(\frac{1}{[K]^n \{K\}} \right)^{\frac{1}{n^2-1}} ds,$$

quand

$$|[K]^{-n}\{K\}^{-1}| < \frac{A}{|s^b|^{n^2}}, \text{ où } A > 0,\ A = \text{const},\ b > 1.$$

Remarque. On voit sans peine que les équations

$$f(x) = \sum_{k=1}^{n} \int_{-\infty}^{+\infty} \ldots \int K_k(x + y_1 + \ldots + y_k)\, u(y_1) \ldots u(y_k)\, dy_1 \ldots dy_k \equiv \mathfrak{R}(x; u)$$

et

$$u(x) = \mathfrak{R}(x; u) + f(x)$$

se réduisent à des équations algébriques au moyen des noyaux $e^{i\xi x}$ et $e^{-i\xi x}$ pourvu que les conditions analogues à (5) et (10) des théorèmes démontrés soient vérifiées.

Il est facile d'etendre la méthode exposée au cas de noyaux de forme plus générale, par exemple, les noyaux $K_k(\varepsilon_0 x + \varepsilon_1 y_1 + \ldots + \varepsilon_k y_k)$ ou bien $K_k(x^{\varepsilon_0} y_1^{\varepsilon_1} \ldots y_k^{\varepsilon_k})$, où $\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_k$ sont ou bien $+1$ ou bien -1.

Institut des mathématiques et de mécanique.
Université de Leningrad.

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MATHÉMATIQUES

SUR LE SCHÈME STOCHASTIQUE DE L'ÉQUATION DE PEARSON

Par A. CHEPELEWSKIJ

(Présenté par S. N. Bernstein, de l'Académie, le 17. II. 1938)

Dans cette note, en suivant les idées de Pearson, je propose un schème stochastique tout élémentaire, correspondant à l'équation aux différences finies de Pearson dans son aspect le plus général.

J'ai montré [1] que toute la variété des courbes, contenues dans l'équation différentielle de Pearson, se trouve déjà dans l'équation aux différences finies (avant le passage aux dérivées).

Supposons que nous avons n facteurs, déterminant une quantité x de sorte que, si ν facteurs ont agi et $n-\nu$ n'ont pas agi:

$$x=\nu.$$

La probabilité d'agir pour le premier est p, $(1-p=q)$. Si k_1 facteurs ont agi et l'ont sait que k_2 n'ont pas agi $(k_1+k_2=k)$, la probabilité d'agir pour le $k+1$-ième sera p_k, $(1-p_k=q_k)$.

Si le $k+1$-ième facteur a agi, on a:

$$p_{k+1}=f(k)\,\varphi(k_1)\,p_k, \qquad q_{k+1}=f(k)\,q_k, \tag{1}$$

S'il n'a pas agi, à force de symétrie:

$$p_{k+1}=f(k)\,p_k, \qquad q_{k+1}=f(k)\,\psi(k_2)\,q_k. \tag{2}$$

Il est facile de voir, que la probabilité y_x pour que la quantité x ait une valeur donnée est

$$y_x=\frac{n!}{x!\,(n-x)!}p^x q^{n-x} f(0)^{n-1} f(1)^{n-2}\dots f(n-2)\,\varphi(0)^{x-1}\,\varphi(1)^{x-2}\dots$$
$$\cdot\,\varphi(x-2)\,\psi(0)^{n-x-1}\psi(1)^{n-x-2}\dots\psi(n-x-2). \tag{3}$$

Pour le cas du schème des balles qu'on ne remet pas dans l'urne:

$$f(k)=1+\frac{1}{N-1-k}, \quad \varphi(k_1)=1-\frac{1}{N_p-k_1}, \quad \psi(k_2)=1-\frac{1}{N_q-k_2},$$

où N est le nombre des balles dans l'urne, N_p et N_q — les nombres des balles blanches et noires.

Nous avons de la formule:

$$\frac{y_{x+1}-y_x}{y_{x+1}+y_x}=\frac{(n-x)\,p\varphi(0)\,\varphi(1)\dots\varphi(x-1)-(x+1)\,q\,\psi(0)\,\psi(1)\dots\psi(n-x-2)}{(n-x)\,p\varphi(0)\,\varphi(1)\dots\varphi(x-1)+(x+1)\,q\psi(0)\,\psi(1)\dots\psi(n-x-2)}. \tag{4}$$

Les fonctions f, φ et ψ ne peuvent pas être données arbitrairement. Pour toutes les valeurs de n et x elles vérifient la relation:

$$[p\varphi(0)\varphi(1)\dots\varphi(x)+q\psi(0)\psi(1)\dots\psi(n-x)]f(0)f(1)\dots f(n)=1. \quad (5)$$

Posons:

$$\varphi(0)\varphi(1)\dots\varphi(x)=\Phi(x+1),\quad \psi(0)\psi(1)\dots\psi(n-x)=F(n-x+1),$$
$$f(0)f(1)\dots f(n)=\Theta(n+1).$$

Alors on tire de (4) et (5):

$$[p\Phi(x+1)+qF(n-x)]\Theta(n)=1, \quad (6)$$

$$\frac{y_{x+1}-y_x}{y_{x+1}+y_x}=\frac{(n-x)\,p\Phi(x)-(x+1)\,qF(n-x-1)}{(n-x)\,p\Phi(x)+(x+1)\,qF(n-x-1)}. \quad (7)$$

Il suit de (6):

$$qF(n-x-1)=\frac{1}{\Theta(n-1)}-p\Phi(x)=\frac{1}{\Theta(n)}-p\Phi(x+1)=\dots$$

Donc

$$p[\Phi(x+1)-\Phi(x)]=\frac{1}{\Theta(n)}-\frac{1}{\Theta(n-1)},$$

où

$$\Delta\Phi(x)=\frac{1}{p}\Delta\left[\frac{1}{\Theta(n-1)}\right]=\frac{\gamma}{p}, \quad (8)$$

$$\Phi(x)=\frac{\gamma x+a}{p}, \quad (9)$$

où γ et a ne dépendent pas de x et n.

De (6) nous avons d'une manière analogue:

$$F(n-x)=\frac{\gamma(n-x)+b}{q}, \quad (10)$$

$$\Theta(n)=\frac{1}{\gamma(n+c)}, \quad (11)$$

d'où

$$f(k)=1-\frac{1}{k+\frac{c}{\gamma}+1},\quad \varphi(k_1)=1+\frac{1}{k_1+\frac{a}{\gamma}},\quad \psi(k_2)=1+\frac{1}{k_2+\frac{b}{\gamma}}. \quad (12)$$

En substituant (9), (10) et (11) au (5), nous aurons

$$a+b=c-\gamma. \quad (13)$$

L'équation (7) se transforme en l'équation de Pearson:

$$\frac{\Delta y_x}{y_{x+1}+y_x}=\frac{A-x}{Bx^2+Cx+D};$$

$$\left.\begin{aligned} A&=\frac{an-b+\gamma}{c-2\gamma}, & B&=-\frac{2\gamma}{c-2\gamma},\\ C&=\frac{(2n-3)\gamma-a+b}{c-2\gamma}, & D&=\frac{(a+2\gamma)n+b-\gamma}{c-2\gamma}. \end{aligned}\right\} \quad (14)$$

La solution sera (3) qui se transforme en

$$y_x=\frac{pq}{\gamma}\,\frac{n!}{x!\,(n-x)!}\,\frac{\Gamma\left(\frac{c}{\gamma}+2\right)\Gamma\left(\frac{a}{\gamma}+x\right)\Gamma\left(\frac{b}{\gamma}+n-x\right)}{\Gamma\left(\frac{c}{\gamma}+n\right)\Gamma\left(\frac{a}{\gamma}+2\right)\Gamma\left(\frac{b}{\gamma}+2\right)}. \quad (15)$$

Les quantités a, b et γ sont presque arbitraires et l'on peut les choisir de façon que B, C et D prennent des valeurs arbitraires.

En particulier a, b et γ peuvent être négatives.

Les seules restrictions sont les conditions:

$$0 \leqslant y_x \leqslant 1 \qquad \sum_{x=0}^{n} y_x = 1$$

que nous n'examinerons pas, parce qu'il est facile de les satisfaire, sans imposer à a, b et γ des restrictions spéciales.

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Leningrad.

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MATHÉMATIQUES

SUR LES CRIBLES DÉFINISSANT DES ENSEMBLES MESURABLES *B*

Par L. KELDYCH

(Présenté par I. M. Vinogradow, de l'Académie, le 19. II. 1938)

Le but de cette note est d'étudier la structure des cribles élémentaires définissant des ensembles mesurables *B*. Préalablement nous sommes obligés d'étudier des cribles mesurables *B* plus généraux.

Nous appelons «peigne» un ensemble formé de segments appartenant à un rectangle, égaux et parallèles à l'un de ses côtés. Nons appelons «crible peigné» Γ un système de peignes dont les segments sont parallèles à l'axe OY et tel que 1) chaque peigne est contenu à l'intérieur d'un nombre fini d'autres au plus; 2) deux peignes etant donnés—deux cas sont possibles: l'un de ces peignes est situé à l'intérieur de l'autre ou bien les deux peignes n'ont aucun point commun. Nous dirons qu'un peigne est de rang k s'il est situé à l'intérieur de $k-1$ peignes du système, et nous désignons la somme de tous les peignes de rang k par S_k. L'ensemble E situé dans l'espace de Baire I_x est défini au moyen du crible Γ, s'il est la projection orthogonale dans I_x de l'ensemble $\Theta = S_1 \cdot S_2 \ldots S_k \ldots$

Nous appelons «chaîne» une suite infinie de peignes du crible Γ telle que chaque peigne est continu dans le précédent. Nous appelons une chaîne «vide», si tous les éléments de la chaîne n'ont aucun point commun. Nous appelons un crible Γ α-peigné et nous le désignons par Γ_α, si les peignes de Γ sont des éléments mesurables B^* de classes α', α étant le plus petit des nombres finis ou transfinis, supérieur à tous les α'.

Nos recherches sont basées sur la proposition suivante:

Lemme fondamental. *Soit Γ_α un crible α-peigné qui définit l'ensemble E. En décomposant chaque peigne en une infinité dénombrable au plus de peignes et en supprimant peut être quelques uns de ces nouveaux peignes on peut obtenir un nouveau crible α-peigné définissant le même ensemble E et tel, que quelle que soit la chaîne de ce crible, il existe un point appartenant à tous les peignes de cette chaîne.*

On démontre ce lemme par recurrence transfinie (4).

Nous appliquons ce lemme aux éléments mesurables B. Soit E un élément de classe α, $E = \prod_{k=1}^{\infty} \sum_{n_1 \ldots n_k} E_{n_1 n_2 \ldots n_k}$, $E_{n_1 n_2 \ldots n_k} = \text{él}\,\alpha'$, $\alpha' < \alpha$. Nous

* On appelle un ensemble mesurable B élément de classe α, él α, s'il est la partie commune d'une infinité dénombrable d'ensembles B de classes inférieures à α. Voir (1).

appelons le système de tous les $\{E_{n_1 n_2 \ldots n_k}\}$ système générateur de E. D'après notre lemme nous pouvons supposer que chaque chaîne de ce système définit un point et un seul de E. Dans ce cas nous dirons que le système $\{E_{n_1 n_2 \ldots n_k}\}$ est régulier. Nous observons que E, à un ensemble dénombrable de points près, est obtenu à partir des $E_{n_1 n_2 \ldots n_k}$ de la même manière que l'espace de Baire I_x est obtenu à partir des portions de Baire. Par cette raison nous considérons l'élément E comme un espace analogue à l'espace de Baire. Alors toute la théorie descriptive des ensembles dans I_x peut être transportée dans E. Nous appelons α-portion de E l'ensemble $E \cdot E_{n_1 n_2 \ldots n_k}$ et nous disons que la classe relativement à E d'un ensemble $\mathcal{E}$ mesurable B contenu dans E est β, $\mathrm{cl}_{\mathcal{E}}(E) = \beta$, si $\mathcal{E}$ peut être obtenu à partir des α-portions de E au moyen de β opérations successives de passage à la limite. Si $\mathcal{E}$ est un élément de classe β relativement à E, nous écrivons: $\mathcal{E} = \text{él}\,\beta(E)$; si α est un él $\beta(E)$ ou un ensemble de classe $< \beta$ relativement à E nous écrivons: $\mathcal{E} \leqslant \text{él}\,\beta(E)$; si $\mathcal{E}$ est fermé relativement à E, $\mathcal{E} = F(E)$, etc. On doit remarquer, que $\mathrm{cl}_{\mathcal{E}}(E)$ dépend essentiellement du système générateur de E: en changeant ce système on change $\mathrm{cl}_{\mathcal{E}}(E)$. Nous démontrons que

Pour qu'un ensemble $\mathcal{E}$ situé à l'intérieur de l'élément E de classe α soit un él$(\alpha+1)$, il est nécessaire et suffisant que, quel que soit le système générateur régulier de E, $\mathcal{E}$ soit au moins un $F_{\sigma\delta}(E)$, si α est un nombre de premier espèce [un $G_\delta(E)$, si α est un nombre de seconde espèce], et qu'il existe un système générateur régulier de E tel que

$$\mathcal{E} = F_{\sigma\delta}(E)\,[\mathcal{E} = G_\delta(E)]\ (^4).$$

Soit maintenant $E = \text{él}\,\alpha$ défini au moyen d'un crible α-peigné ordonné * Γ_α définissant E et tel que les projections dans I_x des peignes de Γ_α coïncident avec les ensembles générateurs de E. *Pour obtenir un crible rectangulaire C définissant E, il suffit de remplacer chaque peigne de Γ_α par le moindre rectangle contenant ce peigne.* Nous dirons que C est le crible correspondant au système générateur donné de E. Si E est un ensemble mesurable B quelconque, on construit facilement un crible analogue en remarquant que E est la somme d'une infinité dénombrable au plus d'éléments (1): $E = \sum_{n=1}^{\infty} E_n$.

Nous appelons le système de tous les ensembles $\{E^n_{n_1 n_2 \ldots n_k}\}$ système générateur de E.

Si x est un point extérieur à E ($x \not\subset E$), nous appelons indice au point x du crible rectangulaire C, $\mathrm{Ind}_x C$, l'indice du crible linéaire correspondant à C au point x.

Nous appelons indice du crible C, $\mathrm{Ind}\,C$, le moindre nombre transfini qui surpasse $\mathrm{Ind}_x\,C$ quel que soit le point x extérieur à E.

Nous désignons d'après N. Lusin l'indice de l'ensemble E mesurable B, ou autrement, l'indice du crible minimum définissant E ** par $\mathrm{Ind}\,E$.

Nous étudions les cribles correspondant aux systèmes générateurs réguliers de tous les ensembles mesurables B, nous définissons les indices

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* On appelle un crible ordonné, si les projections sur l'axe OY des côtés supérieurs des peignes de rang 1 et celles des peignes de rang k contenus dans un même peigne de rang $k-1$ forment une suite simple ascendante de points.

** N. Lusin appelle crible minimum le crible ayant le moindre indice possible parmi tous les cribles définissant E. Voir (2) p. 274 [8].

minima de cas cribles et nous démontrons, que ces indices coïncident avec les indices des ensembles mesurables B correspondants.

Nous introduisons la notion de crible subordonné:

Nous disons qu'un crible rectangulaire C' est subordonné au crible restangulaire C, si C' est formé de rectangles de C (serrés peut être suivant la direction de l'axe OY), déplacés arbitrairement dans la direction positive de l'axe OY, pourvu que dans ce déplacement ils n'entrent pas dans rectangles dont ils ne faisaient pas partie dans C.

Nous démontrons le théorème suivant:

Théorème. *Pour qu'un ensemble $\mathcal{E}$ mesurable B puisse être défini au moyen d'un crible C' subordonné au crible C, correspondant à un système générateur régulier de l'ensemble E mesurable B, il est nécessaire et suffisant que $\mathcal{E}$ fasse partie de E.*

La nécessité est tout à fait évidente. Pour démontrer la suffisance nous pouvons sans troubler la généralité considérer le cas $E = \text{él}\,\alpha$ et nous voyons que la proposition est évidente pour $\mathcal{E} \leqslant G_\delta(E)$. Nous considérons d'abord le cas $\mathcal{E} \leqslant F_{\sigma\delta}(E)$, nous déformons le système générateur de $\mathcal{E}$ en décomposant chaque ensemble générateur en une infinité dénombrable au plus d'ensembles de même classe relativement à E, de manière que le crible C', correspondant au système générateur reconstruit de $\mathcal{E}$, soit subordonné à C. Puis nous appliquons la méthode de recurrence transfinie.

Nous démontrons encore le lemme suivant:

Lemme. *Si le crible rectangulaire ordonné C' est subordonné au crible C définissant E, on a pour chaque point extérieur à E:* $\text{Ind}_x C' \leqslant \text{Ind}_x C$, si l'on a $\text{Ind}_x C = \omega^\alpha$, et $\text{Ind}_x C' \leqslant (\text{Ind}_x C) \cdot m$, si $\text{Ind}_x C = \omega^\alpha \cdot n + \beta$, $\beta < \omega^\alpha$.

On remarque que la proposition est vraie dans le cas $\text{Ind}_x C = n$, et on démontre le lemme en appliquant la recurrence transfinie.

En appuyant sur ce lemme nous démontrons encore quelques propositions sur les cribles correspondant aux systèmes générateurs réguliers des ensembles mesurables B. Comme application de ces propositions nous calculons les indices minima de ces cribles pour tous les ensembles mesurables B. Enfin, en appliquant la quatrième inégalité de M. Sierpinski * pour les constituantes réelles nous démontrons, que ces indices coïncident avec les indices des ensembles mesurables B.

En désignant un élément de classe α par él, un ensemble de classe α accessible inférieurement par $\text{Inf}\,\alpha$ et un ensemble inaccessible de classe α par $\text{Inac}\,\alpha$ et par K_α la classe des ensembles mesurables B de classe α, nous obtenons le tableau suivant **:

$$\alpha > 0 \quad K_{2\alpha} \begin{cases} \text{él} & \text{Ind} = \omega^\alpha \\ \text{Inf} & \text{Ind} = \omega^\alpha + 1 \\ \text{Inac} & \text{Ind} = \omega^\alpha + 1 \end{cases}$$

$$\alpha > 0 \quad K_{2\alpha+1} \begin{cases} \text{él} & \text{Ind} = \omega^\alpha \cdot 2 \\ \text{Inf} & \text{Ind} = \omega^\alpha + 1 \\ \text{Inac} & \omega^\alpha \cdot 2 \leqslant \text{Ind} \leqslant \omega^{\alpha+1} \end{cases}$$

Dans le cas $E = \text{Inac}\,(2\alpha + 1)$ tous les indices donnés dans le tableau peuvent être atteints.

* Voir [3] p. 282.

** Voir [2] p. 276 [10]. Remarquons, qu'en désignant par α^* le plus grand nombre transfini de seconde ordre, inférieur ou égal à α, on obtient $\alpha = \alpha^* + n$, $n \geqslant 0$, et $2\alpha = \alpha^* + 2n$.

Enfin nous démontrons sans difficulté que les bornes supérieures des classes des constituantes réelles d'un complémentaire analytique, que nous avons deja calculées*, sont exactes.

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* Voir (3) p. 284.

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MATHÉMATIQUES

SUR LA RECHERCHE DES SOLUTIONS PÉRIODIQUES D'UNE ÉQUATION DIFFÉRENTIELLE ORDINAIRE DU PREMIER ORDRE PAR LA MÉTHODE DES APPROXIMATIONS SUCCESSIVES

Par N. W. ADAMOFF

(Présenté par S. N. Bernstein, de l'Académie, le 10. II. 1938)

1. Dans une note précédente (1) nous avons étudié quelques propriétés des approximations successives pour l'équation

$$y' = f(x, y) \tag{1}$$

éffectuées à l'aide de l'opérateur

$$A[y] = y(x) + \int_a^x Y\,dx - X\,dy; \quad Y = Xf(x, y).$$

Nous allons montrer maintenant qu'en se servant d'un opérateur semblable on peut trouver les solutions périodiques de l'équation (1). Soit donnée cette équation et soit $f(x, y)$ une fonction continue de x et y dans le domaine $D\,(-\infty < x < +\infty;\ a \leqslant y \leqslant b)$, périodique de période ω:

$$f(x, y) = f(x + \omega, y),$$

admettant dans le domaine D une dérivée continue $\frac{\partial f}{\partial y}$.

Considérons l'opérateur

$$B[y] = y(x) + e^{-kx} \int_0^x e^{kx}[f(x, y) - y']\,dx.$$

Les raisonnements analogues à ceux de la note précédente nous permettent d'établir que, si 1) l'opérateur B a un sens pour les fonctions de la famille $y = \varphi(x, y_0)$ [y_0 est la valeur de la fonction $\varphi(x, y_0)$ pour $x = 0$]; 2) les courbes $y = B[\varphi(x, y_0)]$ sont situées dans le domaine D pour toutes les valeurs de x; 3) les courbes $y = \varphi(x, y_0)$ forment un champ dans le domaine D, c'est-à-dire, deux courbes différentes de la famille n'ont pas de point commun, — les courbes $y = B[\varphi(x, y_0)]$ forment aussi un champ dans le domaine D. C'est une conséquence du fait que la dérivée

$$\frac{\partial}{\partial y_0} B[y] = e^{-kx} \int^x e^{kx} \left(\frac{\partial t}{\partial y} + k \right) \frac{\partial y}{\partial y_0}\,dx + e^{-kx} \left\{ \left(\frac{\partial y}{\partial y_0} \right)_{x=0} \right\}$$

est positive, si la dérivée $\frac{\partial y}{\partial y_0}$ l'est. L'inégalité

$$\frac{\partial}{\partial y_0} B[y] > 0$$

est une conséquence de l'inégalité

$$\frac{\partial y}{\partial y_0} > 0,$$

si le nombre k est positif et assez grand, car alors on a:

$$\frac{\partial f}{\partial y} + k > 0. \tag{2}$$

Nous admettons dans ce qui suit que l'inégalité (2) est vérifiée.

On déduit de cette propriété de l'opérateur B les deux théorèmes suivants.

Théorème I. *Si les courbes*:

$$y = y_0(x);\ y = y_1(x) = B[y_0];\ y = y_2(x) = B[y_1]; \ldots,$$

sont situées dans le domaine D et si l'inégalité

$$y_0(x) > \bar{y}(x)$$

où $y = \bar{y}(x)$ *est une courbe intégrale de l'équation* (1), *située dans le domaine D, est vérifiée pour toutes les valeurs de x, les inégalités*:

$$y_1(x) > \bar{y}(x),\quad y_2(x) > \bar{y}(x), \ldots,$$

sont aussi vérifiées pour toutes les valeurs de x.

Théorème II. *Si l'inégalité*

$$y_1(x) = B[y_0] < y_0(x)$$

est vérifiée pour toutes les valeurs de $x > 0$, *les inégalités*:

$$y_2(x) = B[y_1] < y_1(x);\ y_3(x) = B[y_2] < y_2(x); \ldots,$$

sont aussi vérifiées pour toutes les valeurs de $x > 0$.

(Rappelons, que la démonstration des théorèmes analogues dans la note précédente n'a exigé que l'application de la propriété de l'opérateur, en vertu de laquelle le champ des courbes se transforme à l'aide de l'opérateur en un champ nouveau.)

Il existe des fonctions telles que l'expression $B[y]$ a un sens pour toutes les valeurs de x. La proposition suivante en donne des nombreux exemples: *si la fonction* $y_0(x)$ *est continue dans le domaine D, périodique de période* ω *et telle que* $B[y_0]$ *a un sens pour* $0 \leqslant x \leqslant \omega$, *la fonction* $y_1(x) = B[y_0]$ *est bornée pour toutes les valeurs de x.* En effet, on a pour $0 \leqslant x \leqslant \omega$:

$$y_1(x + n\omega) = y_0(x + n\omega) + e^{-k(x+n\omega)} \int_0^{x+n\omega} e^{kx}[f(x, y_0) - y_0']\,dx = y_0(x) +$$

$$+ e^{-kx}\left\{e^{-kn\omega}\left[\int_0^{n\omega} e^{kx}[f(x, y_0) - y_0']\,dx + e^{kn\omega}\int_0^{x} e^{kx}[f(x, y_0) - y_0']\,dx\right]\right\};$$

en désignant:

$$\int_0^{\omega} e^{kx}[f(x, y_0) - y_0']\,dx = h;\ e^{k\omega} = q$$

et en tenant compte de l'égalité:

$$\int_{n\omega}^{n\omega+x} e^{kx}[f(x,y_0)-y_0']\,dx = e^{k\omega}\int_{(n-1)\omega}^{(n-1)\omega+x} e^{kx}[f(x,y_0)-y_0']\,dx,$$

on a:

$$y_1(x+n\omega) = y_0(x)+e^{-kx}\left\{q^{-n}(h+hq+\ldots+hq^{n-1})+\int_0^x e^{kx}[f(x,y_0)-y_0']dx\right\} =$$

$$= y_0(x)+e^{-kx}\left\{\int_0^x e^{kx}[f(x,y_0)-y_0']\,dx+\frac{h}{q-1}\right\}-\frac{he^{-kx}}{(q-1)\,q^n}$$

et, le nombre q étant supérieur à l'unité,

$$\bar{y}_1(x) = \lim_{n\to\infty} y_1(x+n\omega) = y_0(x)+e^{-kx}\left\{\int_0^x e^{kx}[f(x,y_0)-y_0']\,dx+\frac{h}{q-1}\right\}.$$

Il est facile de voir, que la fonction $\bar{y}_1(x)$ est périodique:

$$\bar{y}_1(x+\omega) = y_0(x)+e^{-kx}\left\{\frac{h}{q}+\int_0^x e^{kx}[f(x,y_0)-y_0']\,dx+\frac{h}{q(q-1)}\right\} = \bar{y}_1(x).$$

2. Nous allons maintenant démontrer le

Théorème III. *S'il existe dans le domaine D une solution $\bar{y}(x)$ de l'équation* (1), *périodique de période* ω, *et si l'on connaît une fonction $y_0(x)$, périodique de même période, telle que l'on a:*

$$y_0(x) > \bar{y}(x)$$

pour $0 \leqslant x \leqslant \omega$ *et*

$$y_0(x) > y_1(x) = B[y_0]$$

pour $0 < x \leqslant \omega$, *la solution périodique peut être trouvée par des approximations successives.*

La dernière condition du théorème est satisfaite, par exemple, quand les courbes intégrales traversent la courbe $y = y_0(x)$ de telle façon, que l'expression $f(x,y)-y_0'$ ne change pas de signe le long de la courbe $y = y_0(x)$. C'est à ce cas qu'on peut souvent réduire par un changement des variables le problème de la recherche des cycles limites de l'équation:

$$\frac{dx}{X(x,y)} = \frac{dy}{Y(x,y)} \qquad (1')$$

dans un domaine annulaire, limité par deux courbes fermées dont l'une est une courbe sans contact. Par exemple: l'équation (1') est telle qu'on a, après l'introduction des coordonnées polaires:

$$\frac{d\rho}{d\theta} = \varphi(\rho,\theta),$$

et la dérivée $\frac{\partial\varphi}{\partial\rho}$ ne devient pas infinie dans le domaine annulaire $\rho_1 \leqslant \rho \leqslant \rho_2$, tandis que le cercle $\rho = \rho_1$ est une courbe sans contact.

En nous servant de l'opérateur:

$$B_1[y] = y(x)+e^{-kx}\int_0^x e^{kx}[f(x,y)-y']\,dx+\frac{he^{-kx}}{q-1},$$

nous trouvons la suite des fonctions

$$\bar{y}_0(x),\ \bar{y}_1(x),\ \bar{y}_2(x),\ldots,\ \bar{y}_n(x),\ldots, \tag{3}$$

où l'on a:

$$\bar{y}_0(x)=y_0(x);\ \bar{y}_{n+1}(x)=B_1[\bar{y}_n]=\bar{y}_n(x)+e^{-kx}\left[\int_0^x e^{kx}[f(x,\bar{y}_n)-\bar{y}'_n]dx+\frac{h_n}{q-1}\right];$$

$$h_n=\int_0^\omega e^{kx}[f(x,\bar{y}_n)-\bar{y}'_n]\,dx;\ q=e^{k\omega}\quad[(n=1,2,\ldots);$$

cette suite est monotone:

$$\bar{y}_{n+1}(x)<\bar{y}_n(x)\qquad(n=0,1,2,\ldots).$$

Pour s'en persuader il suffit évidemment de démontrer que l'inégalité

$$\bar{y}_1(x)<\bar{y}_0(x)$$

a pour conséquence l'inégalité

$$\bar{y}_2(x)<y_1(x).$$

Or, nous avons

$$B[y_1]<y_1(x),$$

où

$$y_1(x)=B[y_0]$$

(voir le théorème II du paragraphe précédent). On a aussi:

$$\bar{y}_1(x)=y_1(x)+\frac{h_0}{q-1};\ h_0=\int_0^\omega e^{kx}[f(x,y_0)-y'_0]\,dx<0, \tag{4}$$

et, d'après le théorème de Green

$$\int_0^\xi e^{kx}[f(x,y_1)-y'_1]\,dx-\int_0^\xi e^{kx}[f(x,\bar{y}_1)-\bar{y}'_1]\,dx+\int_{\bar{y}_1(\xi)}^{y_1(\xi)} e^{k\xi}\,dy-$$
$$-\int_{\bar{y}_1(0)}^{y_1(0)} dy=\iint_{D'}\left(\frac{\partial f}{\partial y}+k\right)e^{kx}\,dx\,dy,$$

où D' est le domaine, limité par les courbes $\bar{y}=y_1(x)$; $y=\bar{y}_1(x)$; $x=0$ et $x=\xi$. Par conséquent, on a:

$$\int_0^\xi e^{kx}[f(x,y_1)-y'_1]\,dx-\int_0^\xi e^{kx}[f(x,\bar{y}_1)-\bar{y}'_1]\,dx=\iint_{D'}\left(\frac{\partial f}{\partial y}+k\right)e^{kx}\,dx\,dy>0,$$

en vertu de l'égalité (4) et de l'inégalité (2). On a d'ailleurs:

$$\int_0^\xi e^{kx}[f(x,y_1)-y'_1]\,dx<0,$$

d'où l'on tire

$$\int_0^\xi e^{kx}[f(x,\bar{y}_1)-\bar{y}'_1]\,dx<0,$$

et enfin:

$$B[\bar{y}_1]<\bar{y}_1(x);\quad B_1[\bar{y}_1(x)]<\bar{y}_1(x).$$

Toutes les fonctions de la suite (3) sont supérieures à $\bar{y}(x)$, où $\bar{y}(x)$ est la solution périodique de l'équation (1). Donc, la suite (3) converge. Désignons sa fonction limite par:

$$Y(x) = \lim_{n\to\infty} \bar{y}_n(x).$$

La suite converge uniformément. On a d'après le théorème de Green:

$$\bar{y}_n(\xi) - \bar{y}_{n+1}(\xi) = \bar{y}_{n-1}(\xi) - \bar{y}_n(\xi) + e^{-k\xi}\left[\iint_{D_n}\left(\frac{\partial f}{\partial y}+k\right)e^{kx}\,dx\,dy + \right.$$

$$\left. + \int_{\bar{y}_{n-1}(\xi)}^{\bar{y}_n(\xi)} e^{k\xi}\,dy + \int_{\bar{y}_n(0)}^{\bar{y}_{n-1}(0)} dy + \frac{h_{n-1}-h_n}{q-1}\right],$$

où le domaine D_n est:

$$0 \leqslant x \leqslant \xi; \quad \bar{y}_n(x) \leqslant y \leqslant \bar{y}_{n-1}(x).$$

En tenant compte de l'égalité

$$h_{n-1} = (q-1)[\bar{y}_n(0) - \bar{y}_{n-1}(0)] \tag{5}$$

on a:

$$\bar{y}_n(\xi) - \bar{y}_{n+1}(\xi) = e^{-k\xi}\left(H_n - \frac{h_n}{q-1}\right),$$

où l'on a:

$$H_n = \iint_{D_n}\left(\frac{\partial f}{\partial y}+k\right)e^{kx}\,dx\,dy;$$

la série:

$$|h_0| + |h_1| + \ldots + |h_n| + \ldots$$

converge, les nombres h_n étant à un facteur constant près égaux aux différences

$$\bar{y}_{n+1}(0) - \bar{y}_n(0).$$

La série

$$\bar{H}_0 + \bar{H}_1 + \ldots + \bar{H}_n + \ldots,$$

où $\bar{H}_n$ est l'intégrale double

$$\bar{H}_n = \iint_{D'_n}\left(\frac{\partial f}{\partial y}+k\right)e^{kx}\,dx\,dy,$$

étendue au domaine D'_n, limité par les courbes:

$$x = 0; \quad x = \omega; \quad y = \bar{y}_n(x); \quad y = \bar{y}_{n-1}(x),$$

converge aussi, la somme:

$$\bar{H}_0 + \bar{H}_1 + \ldots + \bar{H}_n,$$

étant inférieure à l'intégrale

$$\iint_{D'}\left(\frac{\partial f}{\partial y}+k\right)e^{kx}\,dx\,dy,$$

étendue au domaine D', limité par les courbes:

$$x = 0; \quad x = \omega; \quad y = y_0(x); \quad y = \bar{y}(x).$$

En vertu de l'inégalité

$$H_n < \overline{H}_n,$$

on a:

$$0 < \overline{y}_n(x) - \overline{y}_{n+1}(x) < \overline{H}_n + \left| \frac{h_n}{q-1} \right|.$$

Le second membre est le terme général d'une série convergente à termes constants positifs. Donc la suite (3) converge uniformément.

En transformant l'opérateur B_1 à l'aide de l'intégration par parties:

$$B_1[y] = y(0)\, e^{-hx} + e^{-hx}\left[\int_0^x e^{hx}[f(x,y) + ky]\, dx + \frac{h}{q-1}\right],$$

et en tenant compte de ce qu'on vient de démontrer, on voit que la suite des dérivées:

$$\overline{y}_0'(x),\ \overline{y}_1'(x),\ \overline{y}_2'(x), \ldots,$$

converge aussi uniformément. Donc l'égalité

$$Y(x) = Y(x) + e^{-kx}\int_0^x e^{kx}[f(x, Y) - Y']\, dx$$

est vérifiée, c'est-à-dire que la fonction Y est une solution de l'équation (1). Cette solution est périodique comme limite d'une suite uniformément convergente de fonctions périodiques. Cette solution coïncide avec $\overline{y}(x)$ si, bien entendu, il n'existe entre $y_0(x)$ et $\overline{y}(x)$ d'autres solutions périodiques. Il est facile de démontrer (ce que nous omettons ici) que s'il n'éxiste dans le domaine D de solution périodique et si la fonction $y_0(x)$, satisfaisant aux conditions du théorème III, est trouvée, la suite (3) ne peut pas converger dans le domaine D.

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MATHÉMATIQUES

SUR L'EXISTENCE DES SURFACES ALGÉBRIQUES SANS RÉSEAU CONJUGUÉ PERSISTANT

Par N. LUSIN, de l'Academie

1. Les origines de la notion de réseau conjugué persistant. C'est un Mémoire (1) célèbre du géomètre russe K. Peterson sur les relations et affinités entre les surfaces, en 1866, et une Note (2) ultérieure de Ribaucour, en 1891, où, d'une manière indépendante, se trouve mise clairement en lumière la notion de réseau conjugué commun à deux surfaces *, qui furent les origines d'une série de recherches desquelles se dégagea de plus en plus nettement l'importance de la notion de réseau conjugué persistant. Quand une correspondance d'application, point par point, est établie entre deux surfaces S et S' applicables l'une sur l'autre, il existe toujours un réseau conjugué R de S, et un seul, qui se compose de deux familles distinctes **, réelles ou imaginaires, de courbes et qui correspond à un réseau conjugué R' de S' (Peterson, Ribaucour); si un réseau R est conjugué sur trois surfaces S, S' et S'' applicables l'une sur l'autre, il reste conjugué sur une famille infinie continue F de surfaces applicables l'une sur l'autre, qui dépend d'un paramètre arbitraire et qui contient les trois surfaces données S, S' et S'' (Peterson): c'est la déformation à réseau conjugué persistant R [le nom français est dû à Raffy (3), en 1901]. Parmi les géomètres qui contribuèrent à éclaircir et étudier cette déformation, on doit citer les noms de Bianchi, Mlodzieiowski, Voss, Razzaboni, Guichard, E. Cosserat, Raffy, Goursat, Adam, Staeckel, Tzitzeica, Darboux, Egoroff, Demoulin, Servant, Drach, Tachauer, Smith, Eisenhart, Segre, Finikoff, Bucheguennce, Rouyer, Lagally, Terracini, Liebmann, Schur, Freidank, Schaff et plus récemment Gambier, Masloff, Vasseur, Lovett, Rossinski, Lebel, Vincensini.

Soit S une surface donnée; nous désignons par R un réseau de lignes quelconques tracées sur S composé de deux familles distinctes, réelles ou imaginaires.

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* Cette notion fondamentale, que A. Ribaucour croyait nouvelle lorsqu'il l'a donnée dans sa Note insérée le 17 août 1891 aux C. R. de Paris, avait été créée et étudiée antérieurement par K. Peterson, en 1866 (1); la rédaction allemagne (4) a paru en 1868 et la traduction française (5) en 1905.

** En négligeant, bien entendu, les cas spéciaux où les lignes asymptotiques sont des courbes correspondantes: dans ces cas, les surfaces S et S' sont égales ou symétriques, ou bien réglées et leurs génératrices rectilignes se correspondent.

Par définition même, est persistant sur S tout réseau conjugué R tracé sur S tel que, parmi les surfaces applicables sur S, il y a deux surfaces distinctes S' et S'' différentes de S dont les réseaux correspondants R' et R'' provenant de R par l'application de S' et S'' sur S sont conjugués respectivement sur S' et S'' (Peterson).

Si la surface considérée S est donnée en coordonnées curvilignes d'une manière bien déterminée, nous mettons son élément linéaire ds sous la forme $ds^2 = Edu^2 + 2Fdudv + Gdv^2$. Dans ce cas, tout réseau R tracé sur S doit être regardé comme provenant d'un réseau plan r défini par deux équations différentielles:

$$\frac{dv}{du} = \varphi(u, v), \quad \frac{dv}{du} = \psi(u, v) \tag{1}$$

les fonctions φ et ψ étant distinctes et continues ainsi que leurs dérivées partielles des trois premiers ordres; nous désignons par (φ, ψ) ce réseau plan r.

On appelle réseau conjugué persistant de l'élément linéaire donné ds tout réseau plan (φ, ψ) dont le réseau correspondant R sur une surface S qui appartient à ds est persistant [S. Finikoff ([6]) et aussi S. Bucheguennce ([7])].

Maintenant une question très intéressante se pose naturellement: la condition d'analycité d'une surface S est-elle suffisante pour que S ait un réseau conjugué persistant R? Et, si elle ne l'est pas, existe-t-il des surfaces analytiques S' applicables sur S qui le possèdent sûrement?

C'est à tort qu'on est tenté de répondre habituellement à ces difficiles questions par la négative en faisant ce raisonnement simpliste: «Les deux fonctions φ et ψ du réseau conjugué persistant (φ, ψ) de l'élément linéaire ds sont assujetties à trois équations aux dérivées partielles du second ordre

$$\Phi\left(E, F, G, \ldots \middle| \varphi, \psi, \frac{d\varphi}{du}, \ldots, \frac{d^2\psi}{dv^2}\right) = 0, \ \Phi_1 = 0, \ \Phi_2 = 0; \qquad (\Sigma)$$

donc, en général, un élément linéaire n'admet pas de réseau conjugué persistant». En réalité, ce mode de raisonnement n'est plus soutenable dans l'état actuel de la science: les équations aux dérivées partielles $\Phi = 0$, $\Phi_1 = 0$, $\Phi_2 = 0$ sont si compliquées, qu'on ne peut pas songer à démontrer d'une manière précise, en éliminant φ et ψ, l'existence d'une ou de plusieurs relations résultantes $\Omega = 0$, $\Omega_1 = 0, \ldots$ entre les E, F, G et leurs dérivées partielles pour lesquelles on est sûr qu'elles ne sont pas identiquement nulles et, par suite, à établir rigoureusement l'incompatibilité des équations $\Phi = 0$, $\Phi_1 = 0$, $\Phi_2 = 0$ même pour les E, F, G arbitraires. À plus forte raison les difficultés accroissent lorsque nous considérons une famile particulière importante de surfaces telle quelle l'est, par exemple, la famille des surfaces à courbure constante, où les E, F, G ne peuvent pas être pris arbitrairement.

Dans ces conditions, on comprend bien l'attitude de géomètres tels que B. Mlodzieiowski et D. Egoroff que ne considéraient jamais le raisonnement indiqué comme probant. Pour le moment, je me contente de citer deux passages de S. Finikoff qui a insisté fortement à maintes reprises sur la nécessité d'avoir une démonstration précise de l'existence d'une surface analytique sans réseau conjugué persistant: «La question intéressante se pose alors de savoir, s'il existe effectivement une surface entièrement privée des réseaux conjugués persistants, et, d'autre part, s'il existe une surface à une infinité des réseaux conjugués persistants.

La première question attend encore sa solution (1917)» (6). «Nous ne savons aucun exemple effectif qui aurait légitimé ce raisonnement (1937)» (8).

D'ailleurs, il faut distinguer ici les trois problèmes différents:

1) Reconnaître s'il existe une surface analytique ou même algébrique sans réseau conjugué persistant.

2) Reconnaître s'il existe une surface analytique S privée d'un réseau conjugué persistant ainsi que toutes les surfaces analytiques S' applicables sur S.

3) Reconnaître s'il existe une surface analytique S ayant un réseau conjugué persistant ainsi que toutes les surfaces analytiques S' applicables sur S.

Nous allons répondre en affirmative à 1 et 2 et par la négative à 3.

2. Les équations définissant les réseaux conjugués persistants (φ, ψ). Nous commençons par écrire un système d'équations aux dérivées partielles qui sont vérifiées par ces fonctions φ et ψ. De tels systèmes ont été formés: par S. Finikoff (6) de la manière presque explicite et par S. Bucheguennce (7) sous une forme implicite *, tous les deux en 1917.

Nous partons des deux équations aux dérivées partielles de Codazzi

$$\left.\begin{aligned} \frac{\partial \delta}{\partial v} - \frac{\partial \delta'}{\partial u} + p\delta + p'\delta' + p''\delta'' = 0, \\ \frac{\partial \delta''}{\partial u} - \frac{\partial \delta'}{\partial v} + q\delta + q'\delta' + q''\delta'' = 0, \end{aligned}\right\} \tag{2}$$

et de l'équation finie de Gauss

$$\delta\delta'' - \delta^{12} = K, \tag{3}$$

où $p = \left\{\begin{smallmatrix} 22 \\ 2 \end{smallmatrix}\right\}$, $p' = -2\left\{\begin{smallmatrix} 12 \\ 2 \end{smallmatrix}\right\}$, $p'' = \left\{\begin{smallmatrix} 11 \\ 2 \end{smallmatrix}\right\}$, $q = \left\{\begin{smallmatrix} 22 \\ 1 \end{smallmatrix}\right\}$, $q' = -2\left\{\begin{smallmatrix} 12 \\ 1 \end{smallmatrix}\right\}$, $q'' = \left\{\begin{smallmatrix} 11 \\ 1 \end{smallmatrix}\right\}$, K étant la courbure totale de la surface S.

On voit bien que les symboles de Christoffel (à un facteur près) p, p', p'', q, q', q'' et K sont des expressions différentielles dépendant des seules lettres E, F, G et de leurs dérivées partielles. Quant aux δ, δ' et δ'', ce sont les coefficients réduits $\frac{D}{\sqrt{EG - F^2}}$, $\frac{D'}{\sqrt{EG - F^2}}$, $\frac{D''}{\sqrt{EG - F^2}}$ de la seconde forme quadratique $Ddu^2 + 2D'dudv + D''dv^2$ de la surface S. On sait d'ailleurs que la surface S est parfaitement déterminée d'une manière intrinsèque (c'est-à-dire en négligeant la position de S dans l'espace) par la connaissance seule des six fonctions $E, F, G, \delta, \delta', \delta''$ des variables indépendantes u, v; une telle surface sera notée par $S(E, F, G, \delta, \delta', \delta'')$. On sait enfin que la condition nécessaire et suffisante pour qu'un réseau R tracé sur S et défini par un réseau plan (φ, ψ) soit conjugué est que l'égalité:

$$\delta\varphi\psi + \delta'(\varphi + \psi) + \delta'' = 0 \tag{4}$$

soit vérifiée par les fonctions φ et ψ.

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* Il paraît fort probable que c'est la forme implicite du système d'équations définissant (φ, ψ) qui a empêché son auteur de tirer de là l'existence d'une surface analytique sans réseau conjugué persistant. En effet, d'une part, il était impossible, en 1917, de prendre au sérieux le raisonnement simpliste indiqué qui, en fin de compte, n'est qu'une affaire de routine, étant entièrement hors de proportion avec la science contemporaine. D'autre part, l'absence même à cette époque du théorème relatif aux systèmes d'équations aux dérivées partielles que j'ai développé dans ma Note précédente [C. R. Acad. Sci. URSS, XVIII, № 8 (1938)] ne pouvait pas présenter de sensible obstacle à un géomètre de son rang.

Ceci étant, effectuons la substitution *

$$\delta = ax + \frac{b}{x},\ \delta' = -ax\varphi - \frac{b}{x}\psi,\ \delta'' = ax\varphi^2 + \frac{b}{x}\psi^2, \quad (5)$$

où a, b, x, φ et ψ sont des fonctions quelconques des variables indépendantes u et v, deux fonctions a et b étant toujours liées par la relation

$$ab = \frac{K}{(\psi - \varphi)^2}. \quad (6)$$

On s'assure immédiatement que l'equation (3) de Gauss et l'égalité (4) sont vérifiées par δ, δ', δ'' quelles que soient a, b, x, φ et ψ, a et b étant liées par (6). Il résulte de là que si les deux équations (2) de Codazzi sont vérifiées par $\delta, \delta', \delta''$, le réseau R tracé sur $S(E, F, G, \delta, \delta', \delta'')$ et défini par (φ, ψ) sera conjugué. Or, le résultat de la substitution (5) dans les équations (2) de Codazzi est de la forme

$$\frac{\partial X}{\partial u} = A_0X^2 + A_1X + A_2,\ \frac{\partial X}{\partial v} = B_0X^2 + B_1X + B_2, \quad (7)$$

où $X = x^2$, les A et les B étant des expressions différentielles dépendant des seules lettres E, F, G, φ, ψ, a, b et de leurs dérivées partielles. La condition nécessaire pour que le système (7) admette une solution X est que X soit une racine de l'équation du deuxième degré

$$\left\{\frac{\partial A_0}{\partial v} - \frac{\partial B_0}{\partial u} + \left|\begin{matrix} A_0 A_1 \\ B_0 B_1 \end{matrix}\right|\right\}X^2 + \left\{\frac{\partial A_1}{\partial v} - \frac{\partial B_1}{\partial u} + 2\left|\begin{matrix} A_0 A_2 \\ B_0 B_2 \end{matrix}\right|\right\}X + \\ + \left\{\frac{\partial A_2}{\partial v} - \frac{\partial B_2}{\partial u} + \left|\begin{matrix} A_1 A_2 \\ B_1 B_2 \end{matrix}\right|\right\} = 0, \quad (8)$$

ce qu'on peut écrire aussi sous la forme abrégée

$$\Phi X^2 + \Phi_1 X + \Phi_2 = 0. \quad (8^*)$$

Lemme. *Si l'élément linéaire ds est donné, la condition nécessaire et suffisante pour que* (φ, ψ) *soit un réseau conjugué persistant de l'élément linéaire ds est que l'on ait simultanément* $\Phi = 0$, $\Phi_1 = 0$ *et* $\Phi_2 = 0$.

En effet, si (φ, ψ) est un réseau conjugué persistant de l'élément linéaire ds, nous avons une infinité de surfaces $S(E, F, G, \delta, \delta', \delta'')$, sur lesquelles le réseau correspondant R est persistant. Donc, il existe une infinité des solutions distinctes X du système (7); il résulte aussitôt de là que $\Phi = 0$, $\Phi_1 = 0$, $\Phi_2 = 0$. D'autre part, si nous avons $\Phi = 0$, $\Phi_1 = 0$, $\Phi_2 = 0$ l'intégration d'une seule équation de Riccati nous donne une infinité des solutions $X = \frac{C\alpha + \beta}{C\gamma + 1}$ dépendant du paramètre C, les α, β, γ étant des fonctions déterminées des variables u, v. On en conclut que les formules (5) nous donnent une infinité de surfaces S, sur lesquelles le réseau correspondant R est conjugué. Donc, le réseau (φ, ψ) est persistant, ce qu'il fallait à démontrer.

Pour avoir les équations définissant les fonctions φ et ψ sous forme explicite définitive, nous introduisons un opérateur $\Delta_f z$ défini par égalité:

$$\Delta_f z = \frac{\partial z}{\partial v} + f\frac{\partial z}{\partial u}, \quad (9)$$

z et f étant des fonctions quelconques de deux variables indépendantes

* Cf. S. Bucheguennce, Thèse, p. 16 (7).

u, v. Il est à remarquer que cet opérateur jouit des propriétés des fonctions dérivées:

$$\Delta_f(z_1+z_2)=\Delta_f z_1+\Delta_f z_2, \quad \Delta_f(z_1 z_2)=\Delta_f z_1\cdot z_2+z_1\cdot\Delta_f z_2, \quad \Delta_f \log z=\frac{\Delta_f z}{z},$$

et en outre

$$\Delta_g\Delta_f z-\Delta_f\Delta_g z=\frac{\Delta_g f-\Delta_f g}{g-f}\cdot(\Delta_g z-\Delta_f z). \tag{10}$$

Enfin, posons, pour abréger: $p(z)=p-p'z+p''z^2$ et $q(z)=q-q'z+q''z^2$.

Maintenant le calcul détaillé des expressions différentielles Φ, Φ_1 et Φ_2 nous amène aux équations définissant les fonctions φ et ψ des réseaux persistants (φ,ψ) sous la forme explicite définitive:

$$\Delta_\varphi \log\left\{\frac{\Delta_\varphi\varphi-\left|\begin{matrix}p(\varphi)&1\\ q(\varphi)&\varphi\end{matrix}\right|}{K}\right\}+\frac{3\Delta_\varphi\varphi-\Delta_\psi\varphi-2\left|\begin{matrix}p(\varphi)&1\\ q(\varphi)&\psi\end{matrix}\right|}{\psi-\varphi}=0; \tag{I}$$

$$\begin{aligned}&\Delta_\varphi\left\{\Delta_\psi\log\sqrt{K}+\frac{\Delta_\psi\varphi-\Delta_\varphi\psi}{\psi-\varphi}+\frac{\Delta_\psi\psi-\left|\begin{matrix}p(\psi)&1\\ q(\psi)&\varphi\end{matrix}\right|}{\psi-\varphi}\right\}+\\ &+\frac{\Delta_\psi\varphi-\Delta_\varphi\psi}{\psi-\varphi}\cdot\left\{\Delta_\psi\log\sqrt{K}+\frac{\Delta_\psi\varphi-\Delta_\varphi\psi}{\psi-\varphi}+\frac{\Delta_\psi\psi-\left|\begin{matrix}p(\psi)&1\\ q(\psi)&\varphi\end{matrix}\right|}{\psi-\varphi}\right\}+\\ &+\Delta_\psi\left\{\Delta_\varphi\log\sqrt{K}+\frac{\Delta_\psi\varphi-\Delta_\varphi\psi}{\psi-\varphi}-\frac{\Delta_\varphi\varphi-\left|\begin{matrix}p(\varphi)&1\\ q(\varphi)&\psi\end{matrix}\right|}{\psi-\varphi}\right\}+\\ &+\frac{\Delta_\psi\varphi-\Delta_\varphi\psi}{\psi-\varphi}\cdot\left\{\Delta_\varphi\log\sqrt{K}+\frac{\Delta_\psi\varphi-\Delta_\varphi\psi}{\psi-\varphi}-\frac{\Delta_\varphi\varphi-\left|\begin{matrix}p(\varphi)&1\\ q(\varphi)&\psi\end{matrix}\right|}{\psi-\varphi}\right\}+\\ &+2\,\frac{\Delta_\varphi\varphi-\left|\begin{matrix}p(\varphi)&1\\ q(\varphi)&\varphi\end{matrix}\right|}{\psi-\varphi}\cdot 2\,\frac{\Delta_\psi\psi-\left|\begin{matrix}p(\psi)&1\\ q(\psi)&\psi\end{matrix}\right|}{\psi-\varphi}=0;\end{aligned} \tag{II}$$

$$\Delta_\psi\log\left\{\frac{\Delta_\psi\psi-\left|\begin{matrix}p(\psi)&1\\ q(\psi)&\psi\end{matrix}\right|}{K}\right\}-\frac{3\Delta_\psi\psi-\Delta_\varphi\psi-2\left|\begin{matrix}p(\psi)&1\\ q(\psi)&\varphi\end{matrix}\right|}{\psi-\varphi}=0. \tag{III}$$

Dans ce qui suit, nous écrirons le système $(\sum)$ de ces équations sous la forme succincte

$$\Phi=0 \quad (\text{I});\ \Phi_1=0 \quad (\text{II});\ \Phi_2=0 \quad (\text{III}). \tag{$\sum$}$$

Nous allons tirer de ces équations la solution des problèmes posés.

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MATHÉMATIQUES

SUR LE PRINCIPE DE MAXIMUM GÉNÉRALISÉ POUR LES FONCTIONS SUBHARMONIQUES

Par I. I. PRIVALOV

(Présenté par I. M. Vinogradow, de l'Academie, le 25. II. 1938)

Soit D un domaine connexe d'un espace à $p \geqslant 2$ dimensions dont la frontière $\sum$ est composée d'un nombre fini de surfaces jordaniennes fermées et soit E un ensemble arbitraire de la surface $\sum$ de mesure harmonique nulle par rapport au domaine D.

Théorème. *Si la fonction subharmonique $u(P)$ bornée supérieurement dans le domaine D vérifie l'inégalité:*

$$\lim_{P \to Q} u(P) \leqslant C \tag{1}$$

en tous les points Q de la frontière $\sum$ sauf peut être aux points de l'ensemble E, alors on a à l'intérieur de D l'inégalité $u(P) \leqslant C$, le signe d'égalité n'ayant lieu que dans le cas où $u(P)$ est identiquement égal à la constante C.

En partant d'un point arbitraire P_0 du domaine D et d'un $\varepsilon > 0$ aussi petit que l'on veut, on peut construire un ensemble ouvert O de la surface $\sum, O \supset E$, de telle manière que $\omega(O, D, P_0) < \varepsilon$, où l'on désigne par $\omega(O, D, P)$ la mesure harmonique de l'ensemble ouvert O par rapport au domaine D au point P.

Considérons la fonction auxiliaire

$$v(P) = u(P) - C - (K - C)\,\omega(O, D, P),$$

où K est la borne supérieure de la fonction donnée $u(P)$ dans le domaine $D, K > C$ *.

Il est évident que $v(P)$ est une fonction subharmonique dans le domaine D; en ce qui concerne son allure aux points frontières, si Q n'appartient pas à O, on a $\overline{\lim_{P \to Q}}\, v(P) \leqslant 0$ en vertu de (1), et si Q appartient à O, on aura de même $\overline{\lim_{P \to Q}}\, v(P) \leqslant 0$ en vertu des propriétés de la mesure harmonique $\omega(O, D, P)$, dont les valeurs limites aux points de O sont égales à 1.

* Si $K \leqslant C$ le théorème n'exige aucune démonstration.

On a donc pour tous les points frontières de Q:

$$\overline{\lim_{P \to Q}} v(P) \leqslant 0.$$

Par conséquent, $v(P) \leqslant 0$ partout dans D, en particulier $v(P_0) \leqslant 0$, c'est-à-dire:

$$u(P_0) \leqslant C + (K - C)\,\omega(O, D, P_0).$$

En rappelant que $\omega(O, D, P_0) < \varepsilon$, on voit que

$$u(P_0) < C + (K - C)\,\varepsilon,$$

d'où $u(P_0) \leqslant C$, parce que $\varepsilon > 0$ est aussi petit que l'on veut, donc le théorème est démontré.

Si l'on se borne à la considération d'un domaine G borné par une surface du type de Liapounoff S dans un espace à $p \geqslant 3$ dimensions et par une courbe rectifiable de Jordan S dans le cas $p = 2$, on peut démontrer le théorème:

Un ensemble de points E situé sur la frontière S du domaine G est de mesure harmonique nulle par rapport au domaine G dans le cas et dans ce cas seulement, où il est de mesure superficielle nulle.

Cas $p = 2$. Supposons que E est situé sur la circonférence unitaire et qu'il est de longueur nulle. Démontrons que E est de mesure harmonique nulle par rapport au cercle unitaire. En partant d'un $\varepsilon > 0$ aussi petit que l'on veut, construisons sur la circonférence unitaire un ensemble ouvert O tel que $E \subset O$ et mes $O < \varepsilon$.

En désignant par $\omega(E, G, P)$ la mesure harmonique de l'ensemble E par rapport au cercle G au point P, on a:

$$\omega(E, G, P) \leqslant \omega(O, G, P) = \frac{1}{2\pi} \int_O \frac{1 - r^2}{1 - 2r\cos(\theta - \varphi) + r^2},$$

ou r, φ sont les coordonnées polaires du point P. Il suit évidemment de la dernière inégalité:

$$\omega(E, G, P) < \frac{1}{2\pi} \frac{1 + r}{1 - r} \varepsilon,$$

d'où il résulte que $\omega(E, G, P) = 0$ en chaque point de P, ce qu'il fallait démontrer.

Si E est situé sur une courbe rectifiable de Jordan, bornant le domaine G, et possède une longueur nulle, il est de mesure harmonique nulle par rapport à G. La démonstration de ce fait se réduit immédiatement au cas considéré si l'on fait une représentation conforme du domaine G sur le cercle unitaire et si l'on remarque que la longueur de l'ensemble transformé E_1 est encore nulle et que d'ailleurs

$$\omega(E, G, P) = \omega(E_1, G_1, P_1),$$

où G_1 est le cercle unitaire, tandis que P et P_1 sont des points correspondants dans la représentation conforme.

La proposition inverse peut être aussi démontrée, seulement pour le cercle unitaire. Supposons donc que E est situé sur la circonférence unitaire et qu'il est de mesure harmonique nulle par rapport au cercle unitaire. Montrons qu'il est de longueur nulle.

En effet, en partant d'un point P_0 et d'un $\varepsilon > 0$ arbitraire, construisons un ensemble ouvert O sur la circonférence unitaire de manière à avoir $E \subset O$ et $\omega(O, G, P_0) < \varepsilon$.

D'autre part, on a:

$$\omega(O, G, P_0) = \frac{1}{2\pi} \int_O \frac{1 - r^2}{1 - 2r\cos(\theta - \varphi) + r^2}\, d\theta,$$

ou r, φ sont les coordonnées polaires du point P_0.

On conclut de la dernière formule que

$$\omega(O, G, P_0) > \frac{1}{2\pi} \frac{1 - r}{1 + r} \operatorname{mes} O,$$

donc $\operatorname{mes} O < 2\pi \frac{1 + r}{1 - r} \varepsilon$, d'où $\operatorname{mes} E = 0$.

Cas $p \geqslant 3$. En raisonnant d'une manière analogue, nous aurons:

$$\omega(E, G, P) \leqslant \omega(O, G, P) = \frac{\Gamma\left(\frac{p}{2}\right)}{2(p-2)\pi^{\frac{p}{2}}} \int_O \frac{\partial g(P; Q)}{\partial N}\, d\sigma,$$

d'où

$$\omega(E, G, P) < C, \operatorname{mes} O < C\varepsilon,$$

parce que

$$\frac{\partial g(P; Q)}{\partial N} < C, \quad \operatorname{mes} O < \varepsilon;$$

par conséquent,

$$\omega(E, G, P) = 0.$$

Inversement, de la formule

$$\omega(O, G, P_0) = \frac{\Gamma\left(\frac{p}{2}\right)}{2(p-2)\pi^{\frac{p}{2}}} \int_O \frac{\partial g(P_0; Q)}{\partial N}\, d\sigma > c \operatorname{mes} O,$$

puisque

$$\frac{\partial g(P_0; Q)}{\partial N} > c,$$

on a

$$\operatorname{mes} O < \frac{1}{c} \omega(O, G, P_0) < \frac{\varepsilon}{c},$$

car

$$\omega(O, G, P_0) < \varepsilon,$$

donc $\operatorname{mes} E = 0$.

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MÉCANIQUE CÉLESTE

SUR LE DÉVELOPPEMENT DE LA MÉTHODE POUR ÉVALUER LA PÉRIODE D'UNE SOLUTION DU PROBLÈME RESTREINT DES TROIS CORPS

Par N. REIN

(Présenté par V. G. Fesenkov, de l'Académie, le 28. II. 1938)

Considérons le problème restreint des trois corps dans le plan (x, y) rapporté aux axes rotants, dont les équations

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= U'_x, \\ \ddot{y} + 2n\dot{x} &= U'_y, \end{aligned}\right\} \quad n = \text{const}, \quad U = U(x, y), \tag{1}$$

admettent l'intégrale de Jacobi

$$\dot{x}^2 + \dot{y}^2 = 2(U + h), \tag{2}$$

où U est la fonction des forces et h la constante de Jacobi.

Introduisons l'intégrale de Whittaker (1):

$$I = \frac{1}{2\pi} \int\int \Delta^2 \ln (U + h)\, dx\, dy, \tag{3}$$

où Δ^2 est l'opérateur de Laplace et où l'intégration est étendue à la partie du plan (x, y) bornée par une ligne fermée simple et régulière. Soit B une trajectoire fermée simple et régulière du problème considéré et soient C_i et C_e respectivement la borne intérieure et extérieure de la région annulaire renfermant la trajectoire. Supposons qu'a l'intérieur de la région (C_i, C'_e) il n'y a pas de singularités de la fonction des forces, et il n'y a pas de points de la courbe de vitesse nulle:

$$U + h = 0 \tag{4}$$

construite pour la valeur de h correspondant à la trajectoire B.

En désignant par L_B, L_{C_i}, L_{C_e} les valeurs de l'intégrale curviligne

$$L = \frac{1}{2\pi} \int \frac{U'_x dy - U'_y dx}{U + h}, \tag{5}$$

prise le long de B, C_i, C_e, nous avons les formules:

$$I_{BC_i} = L_B - L_{C_i} \tag{6}$$

et

$$I_{BC_e} = L_{C_e} - L_B, \tag{7}$$

où I_{BC_i} et I_{BC_e} sont respectivement les valeurs de l'intégrale (3) étendue aux parties du plan (x, y), renfermées entre les lignes B et C_i et entre les lignes C_e et B.

Ayant égard à ce que L_B est lié avec la période T de la trajectoire B par la relation (2):

$$L_B = -2 \mp \frac{2nT}{\pi}, \tag{8}$$

où le signe supérieur (—) correspond au cas d'une trajectoire du mouvement synodique direct, et où le signe inférieur (+) correspond au cas d'une trajectoire du mouvement synodique rétrograde, nous concluons que la formule (6) et la formule (7) nous donnent la possibilité de la détermination de ladite période T. Dans les cas où la trajectoire B n'est pas donnée et où nous connaissons seulement les bords de la région (C_i, C_e), les formules (6) et (7) peuvent nous fournir une évaluation approchée de la période T, car, vu les hypothèses faites, les termes I_{BC_i} et I_{BC_e} tendent vers zéro quand C_i et C_e tendent vers la trajectoire B.

Dans la note précédente (3) nous avons introduit une courbe auxiliaire dite séparante Φ°, dont l'équation est:

$$\Phi \equiv (U''_{xx} + U''_{yy})(U + h) - (U'^2_x + U'^2_y) = 0. \tag{9}$$

Si la séparante construite pour la même valeur de h que la trajectoire B n'a pas de points communs avec la région (C_i, C_e), alors nous avons ou bien

$$I_{BC_i} > 0\,;\, I_{BC_e} > 0, \tag{10}$$

ou bien

$$I_{BC_i} < 0\,;\, I_{BC_e} < 0, \tag{11}$$

d'où viennent les inégalités

$$L_{C_e} > L_B > L_{Ci} \tag{12}$$

pour le cas (10), et les inégalités:

$$L_{C_i} > L_B > L_{C_e} \tag{13}$$

pour le cas (11), permettant d'obtenir une borne supérieure et inférieure de la période T.

Maintenant nous voulons considérer le cas, où la séparante Φ° a des points communs avec la région (C_i, C_e): nous supposons à cet effet que la séparante (9) coupe la zone (C_i, C_e) un nombre fini de fois en la partageant en régions Φ^+, où $\Phi > 0$, et Φ^-, où $\Phi < 0$. Les branches de la séparante sur lesquelles Φ s'annule en conservant son signe dans le passage d'un côté de la branche à l'autre peuvent, cela s'entend, être exclues de la considération.

Désignons par L_{Φ° la valeur de l'intégrale (5) prise le long de tout les segments de la séparante renfermés à l'intérieur de la zone (C_i, C_e) et prenons pour la direction positive du circuit de la séparante la direction pout laquelle la région Φ^+ reste à gauche. Introduisons ensuite des notations suivantes:

$I^+_{BC_i}[I^+_{BC_e}]$ — la valeur de l'intégrale (3) étendue à toutes les parties de la zone $(B, C_i)[(B, C_e)]$, qui se trouvent dans la region Φ^+;

$I^-_{BC_i}[I^-_{BC_e}]$ — la valeur de l'intégrale (3) étendue è toutes les parties de la zone $(B, C_i)[(B, C_e)]$, qui se trouvent dans la region Φ^-;

$L^+_{C_i}[L^+_{C_e}]$ — la valeur de l'intégrale (5) prise le long de la partie du contour $C_i[C_e]$, située dans la région Φ^+;

$L_{C_i}^-[L_{C_e}^-]$ — la valeur de l'intégrale (5) prise le long de la partie du contour $C_i[C_e]$, située dans la région Φ^-.

Alors nous aurons

$$I_{BC_i}^+ - I_{BC_e}^- = L_B - L_{C_i}^+ - L_{C_e}^- + L_{\Phi^0} \tag{14}$$

et

$$I_{BC_i}^- - I_{BC_e}^- = L_B - L_{C_i}^- - L_{C_e}^+ - L_{\Phi^0}. \tag{15}$$

En ayant égard à ce que

$$I_{BC_i}^+ - I_{BC_e}^- > 0 \tag{16}$$

et

$$I_{BC_i}^- - I_{BC_e}^+ < 0, \tag{17}$$

nous arrivons aux inégalités:

$$L_{C_i}^- + L_{C_e}^+ + L_{\Phi^0} > L_B > L_{C_i}^+ + L_{C_e}^- - L_{\Phi^0}. \tag{18}$$

Les inégalités (18) présentent une généralisation des inégalités (12) et (13) pour le cas où la séparante Φ^0 a des points communs avec la zone, renfermant la trajectoire. Il est aisé de voir que les inégalités (18) restent valables lorsque la séparante a une branche fermée qui est toute entière située à l'intérieur de la zone.

Dans le cas particulier où la zone (C_i, C_e) est entièrement située dans la région Φ^+, nous avons:

$$L_{\Phi^0} = 0;\ L_{C_i}^- = L_{C_e}^- = 0;\ L_{C_i}^+ = L_{C_i};\ L_{C_e}^+ = L_{C_e}, \tag{19}$$

et les inégalités (18) se transforment en inégalités (12). Si au contraire la zone est située entièrement dans la région Φ^-, alors

$$L_{\Phi^0} = 0;\ L_{C_i}^+ = L_{C_e}^+ = 0;\ L_{C_i}^- = L_{C_i};\ L_{C_e}^- = L_{C_e}; \tag{20}$$

et les inégalités (18) deviennent inégalités (13).

À l'aide des inégalités (18) et de la formule (8) nous obtiendrons les inégalités suivantes pour l'évaluation de la période T:

a) Cas de la trajectoire du mouvement synodique direct:

$$-2 - L_{C_i}^+ - L_{C_e}^- + L_{\Phi^0} > \frac{2nT}{\pi} > -2 - L_{C_i}^- - L_{C_e}^+ - L_{\Phi^0}. \tag{21}$$

b) Cas de la trajectoire du mouvement synodique rétrograde:

$$2 + L_{C_i}^- + L_{C_e}^+ + L_{\Phi^0} > \frac{2nT}{\pi} > 2 + L_{C_i}^+ + L_{C_e}^- - L_{\Phi^0}. \tag{22}$$

Les inégalités (21) et (22) permettent d'évaluer la période synodique d'une trajectoire fermée, simple et régulière sans exiger l'intégration des équations du mouvement.

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MÉCANIQUE CÉLESTE

SUR UN SCHÈME SIMPLIFIÉ DU PROBLÈME RESTREINT ELLIPTIQUE DES TROIS CORPS

Par N. REIN

(Présenté par V. G. Fesenkov, de l'Académie, le 28.II.1938)

Considérons le problème restreint des trois corps dans le plan ξ, η. Le mouvement de deux masses finies m_1 et m_2 étant donné, on étudie le mouvement du point matériel soumis à l'attraction de m_1 et m_2. Soient ξ, η les coordonnées de ce point matériel par rapport aux axes fixes dont l'origine coïncide avec le centre d'inertie de m_1 et m_2; soient ξ_i, η_i $(i = 1, 2)$ les coordonnées der masses m_i. Alors on aura les équations du mouvement du point (ξ, η) sous la forme:

$$\ddot{\xi} = U'_\xi; \quad \ddot{\eta} = U'_\eta; \tag{1}$$

où

$$U = \frac{m_1}{r_1} + \frac{m_1}{r_2}, \tag{2}$$

et où

$$r_i = \sqrt{(\xi - \xi_i)^2 + (\eta - \eta_i)^2} \qquad (i = 1, 2). \tag{3}$$

Dans le cas général nous aurons

$$\xi_i = f_i(t); \quad \eta_i = \varphi_i(t) \qquad (i = 1, 2) \tag{4}$$

où f_i, φ_i sont des fonctions connues du temps t.

Dans le cas particulier le plus simple, où les points m_1 et m_2 se trouvent en mouvement uniforme et décrivent des cercles dont le centre commun est le centre d'inertie du système m_1, m_2, Jacobi [1] a démontré l'existence d'une première intégrale des équations (1) qui est bien connue sous le nom de l'intégrale de Jacobi. On sait qu'en introduisant un système d'axes animés d'une rotation uniforme, c'est-à-dire, en faisant le changement des variables:

$$\left.\begin{aligned} \xi &= x\cos nt - y\sin nt, \\ \eta &= x\sin nt + y\cos nt, \end{aligned}\right\} \tag{5}$$

où n est la vitesse angulaire de révolution de m_1 et m_2 autour de leur centre d'inertie, nous ramenons le problème (1) au problème de l'attraction de deux masses fixes dans le champ des forces centrifuges et des

forces de Coriolis. Dans ce dernier problème la fonction des forces U ne contient pas explicitement le temps. On nomme ce cas le plus simple du problème restreint—le «problème restreint circulaire». Dans ce cas nous avons:

$$f_i = (-1)^{i-1} a_i \cos nt; \quad \varphi_i = (-1)^{i-1} \sin nt, \qquad (i = 1, 2) \tag{6}$$

où a_1 et a_2 sont les rayons des orbites circulaires de m_1 et m_2 vérifiant la relation:

$$a_2 = \frac{m_1}{m_2} a_1. \tag{7}$$

Il est à noter que c'est à l'existence de la première intégrale que nous devons plusieurs résultats de l'analyse qualitative des propriétés des trajectoires du problème circulaire.

En supposant que m_1 et m_2 se meuvent le long des ellipses keplériennes, nous arrivons à un cas particulier du problème restreint beaucoup plus compliqué et plus intéressant. C'est le cas du «problème restreint elliptique». Alors f_i et φ_i sont donnés, comme des fonctions de temps, à l'aide des séries de la forme:

$$\left.\begin{aligned} f_i &= (-1)^{i-1} a_i [C_0 + C_1 \cos nt + C_2 \cos 2nt + \ldots + C_k \cos knt + \ldots], \\ \varphi_i &= (-1)^{i-1} a_i [S_1 \sin nt + S_2 \sin 2nt + \ldots + S_k \sin knt + \ldots], \\ & \qquad\qquad\qquad\qquad (i = 1, 2) \end{aligned}\right\} \tag{8}$$

où les coefficients C_k et S_k sont des fonctions connues de l'excentricité e des orbites de m_1 et m_2, et où a_1 et a_2 sont les grands demi-axes.

L'ignorance d'une première intégrale, analogue à celle de Jacobi dans le problème circulaire, présente une grande difficulté pour l'analyse qualitative des propriétés du mouvement dans le problème elliptique. On peut éviter cette difficulté en introduisant un schème simplifié, admettant une intégrale première, analogue à celle de Jacobi et représentant en même temps un pas en avant relativement au problème circulaire.

Nous trouvons dans un travail de Fatou [2] un exemple de construction d'un schème simplifié du problème restreint elliptique. En partant de l'idée de la méthode de Gauss pour le calcul des perturbations séculaires [3], Fatou a construit un schème dans lequel l'attraction des points matériaux en mouvement m_1 et m_2 est remplacée par l'attraction de deux anneaux elliptiques fixes, le long desquels les masses m_1 et m_2 sont distribuées proportionnellement au temps que le point matériel emploie à décrire l'élément ds. Le système des équations du mouvement dans le schème de Fatou admet, sela s'entend, l'intégrale des forces vives. Or, le schème de Fatou ne peut être considéré comme un pas en avant relativement au problème circulaire, car en tenant compte de l'influence moyenne de l'excentricité des orbites des masses perturbatrices, ce schème ne tient compte, en général, que des perturbations séculaires.

Il est à préférer d'introduire un autre schème simplifié. Dans ce schème on remplace par sa partie séculaire non pas toutes les perturbations, mais seulement celles qui sont produites par les déviations du mouvement de m_1 et m_2, du mouvement circulaire et uniforme.

On arrive à ce schème à l'aide des considérations suivantes:

Les équations (1) peuvent être transformées en prenant un système d'axes (x, y) animés de rotation uniforme et ayant son origine O au centre d'inertie de m_1 et m_2; la vitesse angulaire de rotation est égale

au mouvement moyen n de m_1 et m_2. Alors, les équations (1) prendront la forme

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} - n^2 x &= U'_x, \\ \ddot{y} + 2n\dot{x} - n^2 y &= U'_y \end{aligned}\right\} \tag{9}$$

où U est donné par la formule (2), où

$$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} \qquad (i=1,2) \tag{10}$$

avec

$$\left.\begin{aligned} x_i &= f_i \cos nt + \varphi_i \sin nt, \\ y_i &= -f_i \sin nt + \varphi_i \cos nt, \end{aligned}\right\} \quad (i=1,2) \tag{11}$$

et où f_i et φ_i sont donnés par les formules (8).

En remplaçant dans les équations (9) l'expression exacte de U par sa valeur moyenne

$$[U] = \frac{m_1}{2\pi} \int_{nt=0}^{2\pi} \frac{ndt}{r_1} + \frac{m_2}{2\pi} \int_{nt=0}^{2\pi} \frac{ndt}{r_2}, \tag{12}$$

nous arrivons à un schème du mouvement, pour lequel l'attraction des masses m_1 et m_2, se mouvant le long des trajectoires (11), est remplacée par l'attraction des anneaux matériaux, dont les dimensions sont proportionnelles à l'excentricité e des orbites elliptiques et dont les centres se meuvent autour du point O suivant les cercles de rayons a_1 et a_2. La quantité de masse appartenant à l'élément donné de l'anneau est proportionnelle, comme dans la méthode de Gauss, au temps que le point matériel emploie à décrire cet élément. La masse totale de l'anneau est égale à la masse correspondante m_i.

Les équations du mouvement dans le schème introduit peuvent être écrites sous la forme:

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} - n^2 x &= [U]'_x, \\ \ddot{y} + 2n\dot{x} - n^2 y &= [U]'_y, \end{aligned}\right\} \tag{13}$$

et admettent la première intégrale

$$\frac{1}{2}(\dot{x}^2 + \dot{y}^2) = \frac{n^2}{2}(x^2 + y^2) + [U] + h, \tag{14}$$

où h est la constante arbitraire de l'intégration.

En vertu de l'existence de l'intégrale (14) l'analyse qualitative des propriétés du mouvement du schème proposé peut être poussée aussi loin que celle du problème circulaire. Or, l'investigation immédiate des caractéristiques qualitatives du problème est empêchée par cette circonstance que les coordonnées x_i, y_i sont données à l'aide des séries (11). Dans les cas, où il est permis de supposer l'excentricité e suffisamment petite par rapport à l'unité, nous pouvons, toutefois, remplacer les courbes exactes (11), le long desquelles se meuvent m_1 et m_2 dans le système des axes rotants, par des ellipses, dont nous obtiendrons les équations en omettant dans les développements (11) les termes de dégré plus haut que le premier par rapport à e. Dans ce cas nous aurons

$$\left.\begin{aligned} x_i &= (-1)^{i-1} a_i (1 - e\cos nt), \\ y_i &= (-1)^{i-1} 2 a_i e \sin nt \end{aligned}\right\} \quad (i=1,2), \tag{15}$$

donc, le problème est réduit au calcul de l'attraction de l'anneau ellip-

tique de masse m_i, dont le centre est situé à distance moyenne a_i de la masse m_i du centre d'inertie et dont les demi-axes sont $a_i e$ et $2a_i e$.

Les équations du mouvement dans ce dernier problème ont toujours la forme des équations (13), où $[U]$ est représenté comme une somme d'intégrales elliptiques

$$[U]=\frac{m_1}{2\pi}\int_{nt=0}^{2\pi}\frac{ndt}{\sqrt{(x-x_1)^2+(y-y_1)^2}}+\frac{m_2}{2\pi}\int_{nt=0}^{2\pi}\frac{ndt}{\sqrt{(x-x_2)^2+(y-y_2)^2}}. \qquad (16)$$

Ces équations admettent de nouveau l'intégrale première (14).

Dans la présente note nous posons seulement le problème et démontrons l'existence de l'intégrale première (14). Il est à noter que les points de libration de notre problème simplifié doivent rester fixes dans le système des axes rotants et, par conséquent, doivent représenter le mouvement circulaire et uniforme dans le système des axes fixes. Cette circonstance nous conduit à la conclusion que les solutions de libration de notre schème ne correspondent pas exactement aux solutions rigoureuses de Lagrange du problème elliptique restreint, ces dernières devant représenter des trajectoires elliptiques par rapport aux axes fixes. Cela nous amène à la nécessité de l'investigation de la grandeur de cette déviation en fonction de la valeur de l'excentricité e des orbites de m_1 et m_2.

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ASTRONOMY

THEORETICAL ANALYSIS OF THE WOLFF'S METHOD OF STAR COUNTS IN DARK REGIONS

By K. OGORODNIKOFF

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1. The investigation of Schwarzschild's equation of stellar statistics shows that the metod for determining the distance and magnitude of absorption by the curves of $\log B(m)$ and $\log B'(m)$, as proposed by Wolff, can be applied only in the case if: first, the thickness of nebula is negligible as compared to its distance, and second, if the dispersion of absolute magnitudes of stars involved into counts is also very small. In this case absorption and distance can be determined by formula (1):

$$B'(m) = B(m-\varepsilon) + \varepsilon A(m_0), \qquad (1)$$

where $B(m)$ and $B'(m)$ denote the number of stars brighter than mth apparent magnitude on the area of 1 sq. degree in the normal and dark region respectively, ε—absorption in nebula expressed in stellar magnitudes and m_0—the apparent magnitude of stars corresponding to the front boundary of nebula. Practically this case corresponds to counts in dark «spots», i. e. in small, local, sharply outlined dark nebula of the type of «coal sacks». In practice this case corresponds only to counts of stars selected according to the equality of their luminosity.

2. If the thickness of nebula is not small, Wolff's method is not to be applied. In the case of infinitely large nebula, i. e. region within which the percentage of stars contained geometrically inside the volume occupied by the absorbing matter is not small, compared to the number of stars located in front or behind the nebula, the formula:

$$B'(m) = \left(1 + \frac{3}{4}\alpha\Delta m\right) B(m-\Delta m) \qquad (2)$$

takes place provided the dispersion of luminosities is small.

Here α is a constant equal to $1:5\log_{10} e = 0.4605$, and Δm—absorption in stellar magnitudes at the distance corresponding to stars of mth magnitude.

Practically this case corresponds to the counts of stars possessing the same absolute brightness in dark «clouds», i. e. in the obscured regions of large angular extension.

3. For cases when the dispersion of absolute magnitudes of stars is not equal to zero, corrections to Wolff's method for dispersion have been derived.

In case of thin nebula, the formula for determining absorption and distance has the following form:

$$B'(m) = B(m-\varepsilon)+\varepsilon A(m)\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\frac{\rho_0-m-M_0}{\sqrt{2}\sigma}} e^{-\rho^2}d\rho, \qquad (3)$$

where, in addition to known quantities, $A(m)$ denotes the number of stars in the normal region from $m-1$ to mth magnitude, σ—the dispersion of absolute magnitudes, M_0—mean absolute magnitude of stars and ρ_0—logarithmic distance of nebula: $\rho_0 = 5\log r_0$, where r_0 is the distance to the nebula expressed in tens of parsecs.

For the case of thick nebula formula (3) remains valid. Only in this case one determines from formula (2) not the absorption Δm itself, but the quantity $\Delta' m$ connected with Δm by relation:

$$\Delta m = \Delta' m\, e^{-\alpha^2\sigma^2}, \qquad (4)$$

where α and σ have the same meaning as before. Δm refers here to the distance which the stars would have in case their absolute magnitudes were uniform and equal to M_0, $\rho = m - M_0$.

4. When dispersion is large (e. g. when counting all stars together σ is equal approximately to 6) Wolff's method cannot be applied at all. As follows from the analysis of Schwarzschild's equation divergence of Wolff's curves in this case depends chiefly on the character of the luminosity function.

Assume that the apparent distribution of star density satisfies Zeeliger's law:

$$D(r) = D_0 r^{-s},$$

or

$$\Delta(\rho) = D_0 e^{-\alpha s\rho},$$

where s is a certain constant coefficient. Then in the case of thin nebula («dark spot» type) approximate formula takes place

$$\log B'\left(\frac{A}{B}-\frac{A'}{B'}\right) = \log D_0\varphi(m-\rho_0)+\log P, \qquad (5)$$

where ρ_0 is the logarithmic distance of nebula and $P = \alpha\omega e^{\alpha(3-s)\rho_0}\cdot\varepsilon$.

Quantity ω denotes here the angular area of the region under investigation, other quantities being the same as above.

The investigation of the results of counts permits to find quantities s, ρ_0 and ε. First we compute s from the relation which should take place in the case of Zeeliger's law:

$$B(m) = B_0 e^{\alpha(3-s)m}$$

where B_0 is a constant. Practically s can be easily found from the linear formula:

$$\log B(m) = \log B_0 + \frac{3-s}{5}m.$$

Then a curve is drawn for $\log B'\left(\frac{A}{B}-\frac{A'}{B'}\right)$. On this curve a graph of function $\log D_0\varphi(M)$ plotted on tracing paper is superposed so as to make both curves coincide as close as possible, the coordinate axes remaining respectively parallel. Then the shifting of tracing paper in the horizontal direction gives ρ_0 and that in vertical direction $\log P$.

Knowing P it is easy to determine ε.

In the case of thick nebula («dark cloud» type) following formula takes place:

$$\log B' \left(\frac{A}{B} - \frac{A'}{B'} \right) = \log D_0 \Phi (m - \rho_0) + \log P', \qquad (6)$$

where $\Phi(M)$ is the integral luminosity function:

$$\Phi(M) = \int_{-\infty}^{M} \varphi(M)\, dM,$$

and

$$P' = \alpha\omega \frac{\varkappa}{1+\varkappa} e^{\alpha'(3-s)\rho_0},$$

where $\varkappa$ is the logarithmic absorption coefficient. It characterizes the sharpness of the apparent contours of nebula. Formulae (5) and (6) permit thoughever theoretically to determine $\varphi(M)$ provided s, ρ_0 and ε are known.

5. Wolff's curves constructed on the base of star counts often reveal anomalies the most essential of which are:

a) The curve for the obscured regions in the beginning runs sometimes above that for the bright region (intersection of Wolff's curves).

b) Side by side with the initial decrease of the number of stars in the dark region, approximately in 50% of cases, a secondary decrease of this number for fainter stars takes place which, according to the generally adopted Wolff's method, is interpreted as the second dark nebula located behind the primary one («double» nebulae).

c) The depth of the nebulae always proves to be of the same order as the distance to nebula. In other words, all the nebulae have a depth considerably exceeding their transverse dimensions.

The qualitative analysis of Schwarzschild's equation shows that when both the distribution of star density $D(r)$ and of luminosities $\varphi(M)$ in the normal and obscured regions are similar, the intersection of Wolff's curves is impossible.

Yet the intersection is possible if, first, absorption in nebula is not large and, secondly, if the obscured region shows either an excess of general star density, or an excess of absolutely bright stars.

The phenomenon of «double» nebula is probably due to the rapid increase of the luminosity function $\varphi(M)$ at $M > 7$—8 as observed at least in the vicinity of the Sun.

The abnormally large depth of nebulae as derived by Wolff's method can be explained as well by the dispersion of absolute magnitudes of stars.

6. The examination of individual counts generally corroborates the correctness of previous results. Evidently a definitive conclusion requires an investigation of a larger number of nebulae which will be undertaken in the future.

The generalized theory gives the values of absorption and distances which differ considerably from those obtained by Wolff's method. So, for instance, for area № 7 ($\alpha = 20^h13^m$, $\delta = +28°,8$) in Cygnus [Bulletin de l'Observatoire Central à Poulkovo, XIV, 2 (1935)] Wolff's method gives two nebulae with total absorption of 1.7 stellar magnitude, the first of which is located at the distance of 170 parsecs and the second one at 250 parsecs. The generalized method gives a single thin nebula absorbing 2.3 stellar magnitudes and located at a distance of 1000 parsecs.

The analysis of the results of Poulkovo count shows that for obtaining more reliable results one should not confine oneself to small areas of the order of 1 square degree, but take the areas of possibly larger extension. This is especially advisable when counting apparently bright stars, since in small areas their number proves to be exceedingly limited and hence the character of Wolff's curves turns out to be strongly affected by accidental fluctuations of the apparent density of stars. As the experience has shown, 2—3 additional stars can cause a complete alteration in the character of Wolff's curves.

A fuller account of this investigation shall appear in Bulletin de l'Observatoire Central à Poulkovo № 132.

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ASTROPHYSICS

ON THE COMPTON SCATTERING OF RADIATION WITHIN A STAR

By A. B. SEVERNY

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The question as to the rôle of the absorption of radiation due to the Compton scattering by free electrons of stellar matter has been considered by Dirac (1). He has shown that this absorption is small as compared with the photoelectric ones under the conditions predominating in stellar atmospheres. The temperature radiation within the stars possesses wave lengths of no larger than 3 Å, if the stellar matter is perfect gas. With wave lengths of ~ 1 Å the photoelectric absorption will dominate over the absorption due to the Compton effect. However, some late investigations suggest the possibility of the existence within separate stars of regions of matter not obeying the laws of perfect gas; especially of matter possessing a high density of energy ($kT \gg m_0c^2$). Besides, the consideration of the problem on regarding the sources of stellar energy in the aspect of the theory of nuclear transformations leads to the assumption that the radiation within stars has wave lengths considerably smaller than 1 Å. Therefore, to investigate the constitution of such a star, it is important to know the influence of the Compton scattering on the total opacity of stellar matter.

In this paper we consider the following case: at the center of a star there is a region radiating photons with energy $h\nu \gg m_0c^2$; owing to the Compton effect these photons are scattered by free electrons of the matter surrounding this central region. The temperature of this matter is $kT \ll m_0c^2$ and the free electrons are not considered as immovable—their distribution in the phase space is generally submitted to Fermi statistics.

Let φ be the angle made by the direction of the velocity of scattering electron and the direction of the radiation falling upon it with intensity $I(\nu)$. Then owing to the proper motion of the electron with the velocity v, the intensity of the falling radiation will be

$$I'(\nu) = I(\nu) : (1 + \beta \cos \varphi), \tag{1}$$

and frequency:

$$\nu' = \nu (1 - \beta \cos \varphi), \qquad \beta \equiv \frac{v}{c}. \tag{2}$$

If per unit of volume there will be dn electrons, whose velocities are included in the interval $v, v + dv$, the amount of energy, taken off

the incident beam by dn electrons owing to the Compton scattering, will be $\Phi(\nu')\,I'(\nu)\,d\nu\,dn$, where $\Phi(\nu')$ is the total cross section for the Compton scattering, taken per one electron. In case of $h\nu \gg m_0c^2$ for Φ the approximate expression follows from the Klein-Nishina's formula:

$$\Phi \approx \frac{3}{8}\frac{m_0c^2}{h\nu'}\,\Phi_0 \log 2\,\frac{h\nu'}{m_0c^2},\quad \Phi_0 = \frac{8}{3}\pi\left(\frac{e^2}{m_0c^2}\right)^2. \tag{3}$$

The total energy taken off the incident beam of energy $\int I(\nu)\,d\nu$ according to (1), (2) and (3) will be

$$J = \frac{3}{8}\,\Phi_0 \int\int\int \frac{\log\left[\frac{2h\nu}{m_0c^2}(1-\beta\cos\varphi)\right]}{\frac{h\nu}{m_0c^2}(1-\beta^2\cos^2\varphi)}\,I(\nu)\,d\nu\,\frac{8\pi m_0^3}{h^3}\,\frac{v^2dv2\pi\sin\varphi d\varphi}{\frac{1}{A}\,e^{m_0v^2/2kT}+1}, \tag{4}$$

as the electrons are submitted to Fermi statistics. Thus, the coefficient of opacity owing to the Compton effect will be

$$k_c = J : \int I(\nu)\,d\nu. \tag{5}$$

Under the condition that the temperature of matter $kT \ll m_0c^2$, the value of β will be sufficiently small compared with unity, therefore we shall work with the first order of β computing J. With this approximation we have

$$\int_0^{\pi} \log\left[\frac{2h\nu}{m_0c^2}(1-\beta\cos\varphi)\right]\sin\varphi d\varphi \approx 2\log\left[2\,\frac{h\nu}{m_0c^2}e^{-\beta}\right]. \tag{6}$$

The integration over ν in (4) gives

$$F(v) \equiv \frac{2h}{c^2}\int_{\frac{m_0c^2}{h}}^{\infty} \log\left[2\,\frac{h\nu}{m_0c^2}(1-\beta)\right]\nu^2 e^{-\frac{h\nu}{kT_1}}\,d\nu \approx$$

$$\approx \frac{2h}{c^2}\left(\frac{kT_1}{h}\right)^3\left[\log\left(e^q\frac{2kT_1}{m_0c^2}\right)-\beta\right], \qquad q \equiv \frac{3}{2}-B, \tag{7}$$

if we take the Wien approximation for high frequencies $\left(\nu > \frac{m_0c^2}{h}\right)$ (T_1 being the temperature of the central region of star b, Euler's number 0.577), while for the low frequencies $\left(\nu < \frac{m_0c^2}{h}\right)$ we apply the Rayleigh-Jeans' formula. Therefore the integration over the velocities comes to the evaluation of the integral

$$\lambda = \frac{2h}{c^2}\left(\frac{kT_1}{h}\right)^3 \log\left(e^q\frac{2kT_1}{m_0c^2}\right)\left[1-\frac{\varkappa}{c\log\left(e^q\frac{2kT_1}{m_0c^2}\right)}\right]\int_0^{\infty}\frac{v^2dv}{\frac{1}{A}\,e^{\frac{m_0v^2}{2kT}}+1}, \tag{8}$$

$$\varkappa \equiv \int_0^{\infty}\frac{v^3dv}{\frac{1}{A}\,e^{\frac{m_0v^2}{2kT}}+1} : \int_0^{\infty}\frac{v^2dv}{\frac{1}{A}\,e^{\frac{m_0v^2}{2kT}}+1}.$$

If the envelope surrounding the central region of the star represents a gas whose free electrons are not degenerated ($A \ll 1$), then the expression (8) is valid in the form as written. In this case it is easy to evaluate λ, we get:

$$\lambda \approx \frac{2h}{c^2}\left(\frac{kT_1}{h}\right)^3 \log\left(e^q\frac{2kT_1}{m_0c^2}\right)\frac{\sqrt{\pi}}{4}\left(\frac{2kT}{m_0}\right)^{3/2}\times$$

$$\times A\left[1-\frac{2}{\sqrt{\pi}}\left(\frac{2kT}{m_0}\right)^{1/2}\frac{1}{c\log\left(e^q\frac{2kT_1}{m_0c^2}\right)}\right]. \qquad (9)$$

But in case the order of A is equal or more than unity the expression (8) becomes invalid. In this case the degeneracy of gas takes place and we have to draw into account some of the phase space cells already occupied by electrons, i. e. an electron having received a certain impulse from the falling photon is only likely to pass into another cell of the phase space. This probability as determined by the relative numbers of unoccupied states [at the complete degeneracy ($A \gg 1$) being near zero] will be approximately

$$1-\frac{1}{\frac{1}{A}e^{\frac{m_0v^2}{2kT}}+1},$$

so that for $A \geqslant 1$

$$\lambda=\frac{2h}{c^2}\left(\frac{kT_1}{h}\right)^3\int_0^\infty\left[\log\left(e^q\frac{2kT_1}{m_0c^2}\right)-\beta\right]\times$$
$$\times\left(1-\frac{1}{\frac{1}{A}e^{\frac{m_0v^2}{2kT}}+1}\right)\frac{v^2dv}{\frac{1}{A}e^{\frac{m_0v^2}{2kT}}+1}. \qquad (10)$$

The evaluation of integral of (10) at the values of $A \approx 1$ is difficult. In the case of complete degeneracy ($A \gg 1$) the value of λ is near zero, so that the electron gas will be practically transparent for the radiation $h\nu \gg m_0c^2$ provided the absorption of the radiation depends on the Compton effect only. However, it should be noted that in this case when the density of electrons is very high (say, about 10^{30} per 1 cm^3), the mean distance between the electrons (near 10^{-2} Å) may become comparable with the wave length of the scattered radiation. In such a case the amplitudes of the oscillations of two separate fields of adjoining electrons will be added almost arithmetically, so that the energy of a resulting field will be almost twice as large as the energy of the separate field. It follows, that at a high degeneracy of matter such an effect of the interference of separate fields may increase the opacity of matter.

At the temperature of envelope $kT \ll m_0c^2$ we may neglect the whole second term in parentheses of (9). Then according to (12), (4) and (6) after some reductions we get with $A \ll 1$:

$$J=12\Phi_0\pi^2\left(\frac{m_0}{h}\right)^4c^2\lambda=6\pi\Phi_0m_0\left(\frac{kT_1}{h}\right)^3\log\left(e^q\frac{2kT_1}{m_0c^2}\right)n. \qquad (11)$$

Having used for $I(\nu)$ in (5) the Wien approximation we obtain for k_c the following expression

$$k_c=\frac{\pi}{2}\Phi_0\frac{m_0c^2}{kT_1}e^{\frac{m_0c^2}{kT_1}}\log\left(e^q\frac{2kT_1}{m_0c^2}\right)\cdot n. \qquad (12)$$

From (12) it follows: a) at the «temperature» of radiation $kT_1 \gg m_0c^2$ the coefficient of the Compton opacity does not depend on the temperature of the medium the radiation has passed through, b) with the increasing of the penetrating power of radiation (kT_1 increases), k_c decreases (the same is shown by the experiments on the absorption of γ-rays), c) at $kT_1 \gg m_0c^2$ the value k_c (per electron) will be of the order of «classical value» Φ_0, which is the property of γ-rays too. As the number

of free electrons is $n = \frac{\rho}{\mu H (1+f)}$ in the condition of complete ionization (ρ, the density, μ, molecular weight, H, mass of hydrogen atome, f, the number of ions per electron), so designating the product of slow changing factors exp. log by $\psi(T_1)$ we obtain:

$$k_c = \frac{\pi}{2} \Phi_0 \frac{m_0 c^2}{k T_1} \psi(T_1) \frac{\rho}{\mu H (1+f)}. \qquad (13)$$

The comparison with the current photoelectric opacity (as given by Kramers-Eddington) $k_p = 2 \cdot 10^{26} \frac{\rho}{\mu (1+f) T^{\frac{7}{2}}}$ gives

$$\frac{k_c}{k_p} = 9.35 \cdot 10^{-26} \psi(T_1) \frac{T^{\frac{7}{2}}}{T_1}. \qquad (14)$$

(14) shows that the Compton absorption within stars is small compared with the photoelectric one at the temperatures $T < 10^{10}$°K. They become comparable when the temperature of the envelope is of the order of the temperature of the star core and if this temperature is $\sim 10^{10}$ °K and higher. Hence it does not follow that the stellar matter is transparent for the penetrating radiation. For stars in nature the mean density varies from 10^{-5} g/cm³ (supergiant of M-type) till $\sim 10^5$ g/cm³ (white dwarfs). For the wide range of values $m_0 c^2 / k T_1$, the value of k_c will be of order 10^{-25}. Thus, the optical depths τ for the Compton opacity will be: a) for the giants with radii $\sim 10^{12}$ cm $\tau \approx 10^5$, b) for the Sun $\tau \approx 10^9$, and c) for the white dwarfs $\tau \approx 10^2$ provided that their non-degenerate envelopes have each a linear depth $\sim 10^{-1}$ of radius. Thus, generally speaking, the stellar matter is opaque for the penetrating radiation. As it is easy to show for the optical depths $\tau \gg 1$, the time t for the photons to be exuded through the mass of matter with the linear depth r_1, owing to diffusion, will be

$$t = \frac{3}{2} \tau \frac{r_1}{c}. \qquad (15)$$

From (15) it follows that for the white dwarfs the penetrating radiation may exude through the envelope in a part of a second; for giant the time t may reach a year; for our Sun $t \approx 100$ years. If by (15) we determine the time of the diffusion for the photoelectric absorption of the current radiation with $\lambda_{\max} \approx \frac{1}{2} \mu$, then for the Sun this time proves to be of order of 10^5 years, i. e. the value is much greater that for the photons $h\nu \gg m_0 c^2$. It is of interest to note that the optical depths of stars for the absorption of radiation owing to the photoelectric formation of pairs will be of the same order of magnitude as for the Compton absorption.

Therefore, if within the stars penetrating radiation is formed, it practically exudes through the star within a negligible time and comes out into space.

In conclusion we may note that diffuse nebulae with the density $\sim 10^{-22}$ g/cm³ are practically transparent for penetrating radiation.

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PHYSICS

ANGULAR DISTRIBUTION OF PAIRS IN KRYPTON

By L. V. GROŠEV and I. M. FRANK

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1. In our previous paper(1) the probability of pair creation in krypton by γ-rays ThC″ has been discussed. The present article is devoted to the question of the angular distribution of pair components. Angle measurements were made by means of a stereocomparator designed in our laboratory(2). The spatial position of three points was determined: that of the vertex of the pair, of one point on the electron track, and of one on the positron track. From the difference of the coordinates of these points respective angles were calculated, corrections being made for track curvatures produced by the magnetic field.

As has been stated in our previous paper(1), 2—4 cases out of 80 observed may possibly be referred rather to reflections than to pairs. Since a reliable identification of such reflections is impracticable, the whole of the material has been treated. The presence of a small number of reflections cannot essentially distort the result obtained.

The following angles were subjected to measurement: 1) Angle ψ formed by the direction of the photon and that of the emission of the electron. 2) Angle χ formed by the directions of the photon and that of the emission of the positron. 3) Angle φ between the directions of the emission of the electron and the positron. 4) The dihedral angle Φ formed by the planes passing one through the electron and photon, the other through the positron and photon directions.

2. In Table 1 and Fig. 1 the angular distribution for the directions of the emission of the electron and positron (angles ψ and χ respectively) is given. In the last line of the table these two distributions are summarized.

Table 1

Angular range	0—10	10—20	20—30	30—40	40—50	50—60	60—70
Number of electrons	7	16	19	8	13	6	4
Number of positrons	14	18	15	11	12	4	1
Total	21	34	34	19	25	10	5

Table 1

Angular range	70—80	80—90	90—100	100—120	Above 120	Total
Number of electrons	3	0	1	0	0	77
Number of positrons	2	0	0	0	3	80
Total	5	0	1	0	3	157

Data on the angular distribution of pair components had already been obtained by L. Simons and K. Zuber([3]). They did, however, not analyse these data for electrons and positrons separately, giving a common plot for particles of both signs. Their data do mostly agree with those given in Fig. 1, *c*, although their average angle is somewhat smaller than that obtained by us. Simons and Zuber compare their data with Bethe and Heitler's ([4]) theoretical formula for the angular distribution of particles with the full energy E: $\Phi(\Theta)\,d\Theta = \frac{\Theta d\Theta}{(\Theta_0^2+\Theta^2)^2}$, where $\Theta_0 = \frac{mc^2}{E}$.

This formula has been deduced by Bethe and Heitler in Born's approximation under the condition that $E \gg mc^2$. The validity of this formula in the case of pairs from photons with an energy of $5\,mc^2$ is therefore not evident.

The theoretical curve calculated for $E = \frac{h\nu}{2} = 1300$ ekV is plotted in Fig. 1,*c*. It may be seen from this plot that the distribution observed is, at least qualitatively, in good agreement with this curve.

Theoretically, in Born's approximation angular distribution of electrons and positrons should be equal for particles of both signs. However, the two distributions practically somewhat differ, as may be seen from Table 1 and Fig. 1. The difference is particularly conspicuous for small angles. Thus, the angular range from 0 to 15° embraces 34% of the total positrons, and only 18% of electrons. As to the average angles for electrons and positrons, the difference observed is within the limits of measurement errors. The average angle for positrons is equal to $\bar{\chi} = 30°$, and that for electrons to $\bar{\psi} = 33°$. In positrons three cases may be observed when a positron travels at an angle larger than 120° (these cases have not been plotted in Fig. 1). It is possible that these very three pairs are in fact no pairs but reflections of Compton-electrons in gas. If computing the average angles we neglect the pairs which correspond to these positrons, we obtain $\bar{\chi} = 26°$ and $\bar{\psi} = 31°$.

The difference in the angular distribution of electrons and positrons should, obviously, be referred to the influence of the charge of the nucleus. This difference may be partially explained by the imperfectness of pair-selection: in our photographs three cases have been registered when the positron is practically carrying the whole pair energy. As predicted by theory, all such positrons form small angles (smaller than 15°)* with the direction of the photon motion. Reverse cases, when the whole energy of the pair was carried by the electron, have not been observed by us. It is possible that such pairs are present in our photographs. They must, however, have been omitted, since it was difficult to distinguish them from Compton-electrons.

* The energy of the electrons of these three pairs is so small that the angles of emission could not be measured. It is the reason why angles for electrons were measured for 77 pairs instead of 80.

Nevertheless, it may scarcely be supposed that the difference in the angular distribution of electrons and positrons should entirely be explained by this circumstance. Thus, the difference in angular distributions seems to be real; the more so as the charge of the nucleus proves to produce an appreciable effect in krypton.

3. The distribution of electrons and positrons according to energies is given in Fig. 2. From these distributions we obtain as the mean energies for positrons and electrons $\overline{E}_+ = 840$ ekV and $\overline{E}_- = 740$ ekV respectively. Hence $\overline{E}_+ - \overline{E}_- = 100$ ekV*.

The difference of mean energies should be proportional to the atomic number. For lead it was calculated by H. R. Hulme and J. C. Jaeger [5] and

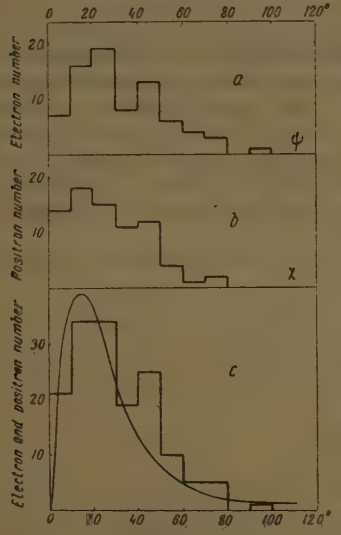

Fig. 1.

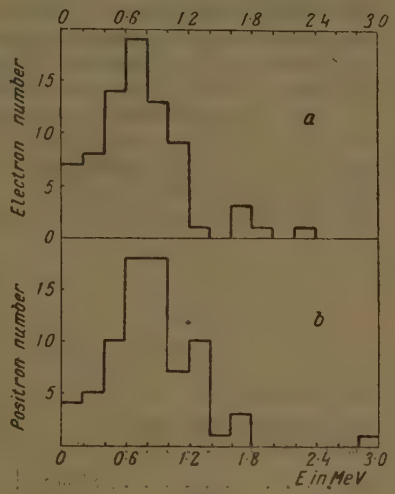

Fig. 2.

proved to be equal to 280 ekV. We should, therefore, expect for krypton $\overline{E}_+ - \overline{E}_- = 120$ ekV. For particles of pairs in argon and iodine this difference has been measured by L. Simons and K. Zuber [3]. Their data are tabulated together with those obtained by us in Table 2.

Table 2

Gas	A	Kr	J
Exper. $\overline{E}_+ - \overline{E}_-$, ekV	10	100	180
Theor. $\overline{E}_+ - \overline{E}_-$	60	120	165

From the table it may be seen that the difference $\overline{E}_+ - \overline{E}_-$ does in fact increase with increasing atomic number, its value agreeing with that theoretically predicted within the limits of probable error.

* As was said above, the conditions of selection of pairs, the energy of which is practically concentrated in one sole particle, are different for positrons and electrons. If we try to equalize the conditions, omitting from the computation of mean energies those pairs in which the whole energy is being carried by the positron, the difference between $\overline{E}_+ - \overline{E}_-$ will not essentially change.

4. In Table 3 and Fig. 3 the distribution for an angle between the direction of electron and positron started is given (angle φ). The distribution does not involve the three pairs, which proved to possess electrons of such slowness that their direction could not be determined (see footnote to § 2).

Table 3

Angular range	0—20	20—40	40—60	60—80	80—100	100—120	Total
Number of pairs	13	19	20	16	7	2	77

The average angle $\bar{\varphi}$ computed from these figures proved to be equal to 48°.

The average angle $\bar{\varphi}$ for pairs in krypton and xenon has been established by W. Bothe and H. Klarmann (6) from a very small statistical material. The average angle found by them for krypton is equal to 19°, that for xenon—to 42°. Hence the conclusion was drawn that this angle increases with the increase of the atomic number. L. Simons and K. Zuber (3) consider this difference as a mere measurement error, and assign the pairs in argon and iodine similar average angles (31°).

All these data are summarized together with our own data in Table 4.

Table 4

Gas	A	Kr	Kr	J	Xe
Number of pairs	18	4	77	24	9
Average angle between positron and electron	30°	19°	48°	32°	42°
Authors	Simons Zuber	Bothe Klarmann	Grošev Frank	Simons Zuber	Bothe Klarmann

As may be seen from this table, the value found by us for the average angle considerably exceeds those obtained by other authors*. As has been said above, among the pairs measured by us a small number of reflections could possibly be present. The average value of the angle φ can, however, not be essentially affected by this fact. On the other hand, this divergence cannot be referred to a mere statistical error.

5. Besides angles ψ, χ and φ, the dihedral angle Φ, connected with them, has also been computed; this angle is formed by two planes, one of which is determined by the direction of the emission of the electron and the direction of the photon; the other, by the direction of the photon and that of the emission of the positron. This angle enters alongside of the angles ψ and χ as a variable in Bethe-Heitler's (4) formula for the differential cross-section of the process of pair-creation.

* At the II All-Union Conference for the study of atomic nucleus, where the preliminary results of our investigations had been reported, data were also presented, obtained by B. C. Jelepov by means of the counter method. He obtained a still larger value for the average angle $\bar{\varphi}$. However, his results cannot be directly compared with ours, since they had been obtained for lead and with different energies of γ-rays (7 MeV).

The distribution for the angle Φ is given in Table 5 and plotted in Fig. 4.

Table 5

Angular range	0—20	20—40	40—60	60—80	80—100	100—120	120—140	140—160	160—180	Total
Number of pairs	4	5	7	7	10	6	11	15	12	77

The average value of $\bar{\Phi}$ is equal to 107°.

If angle $\Phi=180°$, the direction of the photon lies in the plane determined by the directions of the positron and electron (plane of the pair) and is situated between these directions (in this case $\varphi=\psi+\chi$). In case $\Phi=0$, the direction of the photon is also lying in the pair plane, although without the angle φ (here $\varphi=|\psi-\chi|$). As may be seen from the distribution, the most

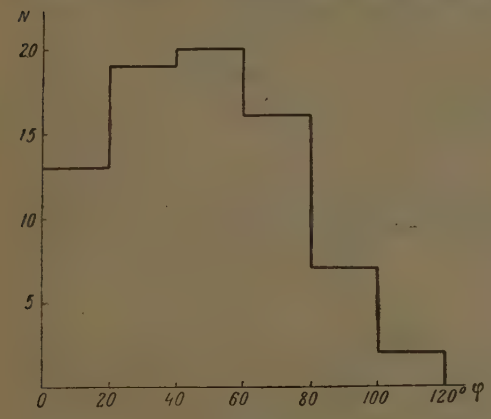

Fig. 3.

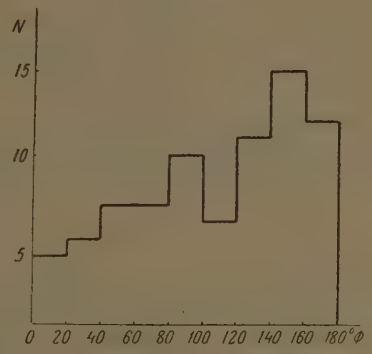

Fig. 4.

probable cases are those approaching to the first one. Indeed, the angle between the photon direction and the plane of the pair has, as a rule, but a small value. It averages about 12°. The direction of the photon if projected on the plane of the pair would in most cases (60 out of 77) fall within angle φ. In 15 cases it would pass the limits of the angle by no more than 12°. As to the remaining two cases, we have already assumed that these are likely to be no pairs, but reflections, their positron forming a large angle with the direction of the beam (more than 120°, see § 2).

The distribution theoretically predicted for all the angles considered here is deduced from the formula for the differential cross section (4). This formula has to be integrated along all the parameters except one, which should be taken for comparison. This would, however, result in most complicated correlations. It is much easier to find the distribution for one parameter, the values for the others being given. Computations of this kind have recently been performed by G. Burkhardt (7). However, no comparison of results thus obtained with our data can be made, since such comparison would necessarily lack statistical accuracy.

P. L. Lebedev Physical Institute.
Academy of Sciences of the USSR.

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GEOPHYSICS

ON A NEW ANOMALOUS EFFECT IN THE SHORT-WAVE END OF THE SOLAR SPECTRUM*. I.

By S. RODIONOV, E. PAVLOVA and N. STOOPNIKOV

(Communicated by S. I. Wawilow, Member of the Academy, 27. II. 1938)

According to modern conceptions the distribution of energy at the short-wave, ultra-violet end of the solar spectrum measured at the earth surface is sufficiently well characterized by the extinction equation of the atmosphere, which has the following form:

$$I_\lambda = I_0 \cdot 10^{-[\alpha_\lambda x\varphi(z)+\beta_\lambda' f(z)+\sigma f(z)]} \qquad (1)$$

where I_λ is the intensity of monochromatic light of wave-length λ ($2850\text{Å} < \lambda < 3\,300\text{Å}$); I_0—the intensity of light of the same wave-length outside the atmosphere; α_λ—the coefficient of absorption of ozone (O_3); x—the thickness of a layer of ozone with normal pressure and temperature; β_λ—the coefficient of Rayleigh scattering; σ—the coefficient of scattering on suspended particles; $f(z)$ and $\varphi(z)$ the so-called Bemporade functions, representing the respective lengths of a ray's path through a layer of ozone [$\varphi(z)$] and through the whole atmosphere [$f(z)$], z being zenith solar distance.

The solution of equation (1) for x is a well known method for determining the total content of ozone (layer thickness), which is the main factor of extinction in this region of spectrum. This method, first suggested by Fabry and Buisson (1) and thoroughly developed by Götz and Dobson (2), has been used by the authors in investigating the short-wave end of the solar spectrum on mount Elbruz in 1935 (3).

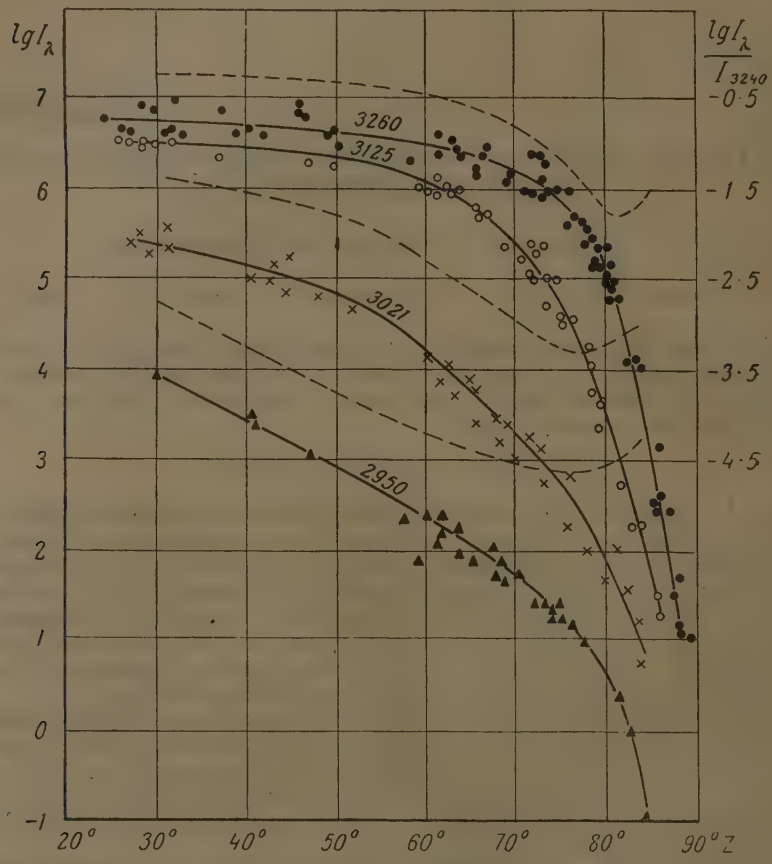
The values of I_λ are determined experimentally, i. e. are measured for different λ; the curves of daylight course $\lg I_\lambda = F(z)$. The steepness of these curves increases as λ grows less, which is in full conformity with equation (1).

Measurements of the short-wave end of the solar spectrum for purposes of the extinction analysis of the stratosphere and in particular of the layer O_3, effected with the help of equation (1), were usually conducted with comparatively small values of z. Considerable interest belongs, however, to extending the sphere of these investigations to greater magnitudes of z, which will provide new information on the absorption of ozone in early morning and late evening hours and will also supply the necessary data regarding the

* Delivered at the meeting of the Stratosphere Committee of the Academy of Sciences of the USSR on the 17th of January, 1938.

applicability of equation (1) to considerable zenith distances (slanting of solar rays in passing through atmosphere). This problem was not yet been solved on account of the insufficient sensitivity of the photometric methods used.

The authors, who formed part of the Fourth Elbruz Complex Scientific Expedition of the Academy of Sciences and the All-Union Institute of Experimental Medicine, undertook an investigation of the solar spectrum at an altitude of 4250 m. (Measurements of such purport are effected at great



altitudes with a view to excluding the detrimental influence of scattering upon suspended particles.) As an objective photometer a counter was used which had been previously devised by one of the authors. The counter was connected to a monochromator of double decomposition, the entrance slit of which was illuminated by the direct rays of sun (the method is described at length in an earlier work of the authors ([3])].

Measurements of the intensity of solar light of four wave-lengths (3260, 3125, 3021 and 2950 Å) were effected simultaneously, beginning with the highest values of z (from sunrise). The figure gives a tabulation of the measurements carried out from the 31st of July to the 15th of August, 1937. The values of lg I_λ in relative units are plotted on the axis of ordinates.

As will be seen from the figure, there exists in the region of z a normal growth of intensity from 25 to 60°; the curve here is the steeper, the smaller z is. Treatment of this part of the curves according to the Fabry and Buisson method (1) gives good straight lines and also a good value for the thickness of the ozone layer, $x=0.25$ cm, which closely approximates the values obtained under the same conditions previously and also bears testimony to the validity of equation (1) for this region of zenith distances.

We have an entirely different picture for the region of z within 60 to 90° (figure). This morning (or evening) course is characterized, in contradistinction to the daylight course, by a change in the intensity of lower wave-lengths: the slope of the respective curves will be the less steep, the lower λ is.

This effect stands out in particularly bold relief if $\lg I_\lambda/I_{3260}$ is plotted instead of $\lg I_\lambda$ (this is done in figure; dotted-line curves). With great zenith distances, magnitude $\lg I_\lambda/I_{3260}$ [difference between ordinates of curves $\lg I_\lambda=F_1(z)$ and $\lg I_{3260}=F_2(z)$] at first declines with growing z (normal daylight course), then reaches its maximum and lastly begins increasing with great magnitudes of z.

The curves of the figure thus evidence that the normal distribution of energy in the short-wave part of the solar spectrum, valid with $z \leqslant 60°$ obtained by us previously (3) in absolute units, with $z > 60—70°$ passes into a certain anomalous distribution, i. e. it would seem, as if atmosphere were becoming more transparent for small values of λ.

The explanation of the effect by increased transparence of the stratosphere for light of small wave-lengths at the expense of smaller contents of O_3 in the morning and evening, is in contradiction with available data on the absence of a daily course of ozone.

We believe that the only possibility of explaining these effects lies in taking account of the possible anomalous refraction of short-wave rays ($\lambda<3200$ Å) in the O_3 layer. The magnitude of this refraction should be abnormally high in the neighbourhood of the long-wave boundary of the Hartley band (anomalous dispersion). Thanks to this, the path of these rays through the O_3 layer must, with given z, be much less than for rays of $\lambda<3200$ Å, which possess a normal refraction. Advancing a purely quantitative theory of the effect seems, however, at the present moment to meet with some difficulties owing to lack of additional experimental data.

All-Union Institute
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GEOPHYSICS

CONTRIBUTION TO THE PROBLEM OF THE UMKEHR-EFFEKT*. II

By S. RODIONOV and E. PAVLOVA

(Communicated by S. I. Wawilow, Member of the Academy, 27. II. 1938)

Götz and Dobson (1) have given a detailed description of the so-called «Umkehr-Effekt» discovered in 1929 in the course of measuring the scattered ultra-violet light of the sky. The effect consists in that, at great zenith distances of the sun, z, the scattered zenith light intensity ratio of two close wave-lengths, in the region of 3300—3100 Å, goes downward with growing z, passes a minimum at a definite z and is growing with further increase of z. This effect was explained by Götz who supposed the light measured by an instrument to be emitted only from a certain effective scattering layer, above which the low density of air makes it possible to take no account of dispersion and below which the small quantity of direct light does the same. The height of the effective scattering layer increases with enhanced zenith distance of sun, z, and with decreasing wave-length. With changes in the magnitude of z at sunset and sunrise, the effective scattering layers of two wave-lengths pass through a layer of ozone at different time: the passage of the effective layer of lower wave-length determines the minimum of the Umkehr-Effekt curve.

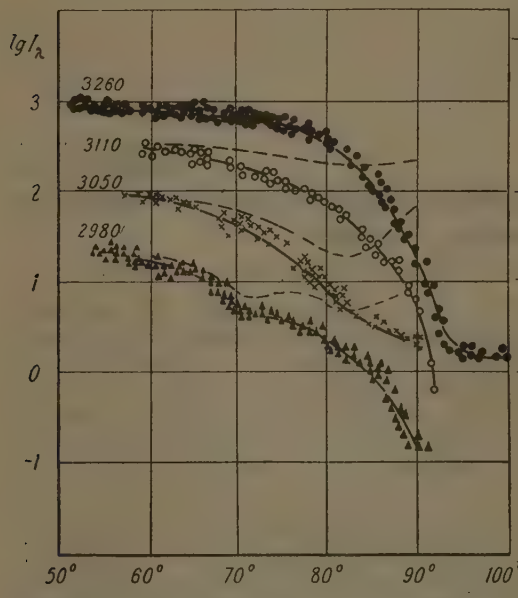
Umkehr-Effekt was observed by Götz and Dobson only for two wave-lengths, 3100 and 3260 Å; these authors made no observations whatever at lower wave-lengths, owing, as it seems, to the insufficient sensitivity of their method of measurement. Götz and Dobson on the basis of the Umkehr-Effekt suggest an approximate method for calculating the distribution of atmospheric ozone.

The authors, while taking part in the 1937 Elbruz expedition, carried out a number of measurements of the zenith light of the sky, making use of high values of z. The method of the light counter was used in these measurements.

The measurements were conducted at an altitude of 2200 m. A monochromator of single decomposition with a breadth of the entrance slit equal to 0.05 mm was used. The number of discharges in the counter, proportional to the intensity of light, was automatically recorded. The intensity of light of two wave-lengths was measured alternately. For working with a broad range of intensities special weakening filters were resorted to. The great sensitivity of the method (intensities of the order of 1000 quants/sec. cm^2 could be measured) permitted of observing the Umkehr-Effekt for wave-lengths up to $\lambda = 2980$ Å incl.

* Delivered at the meeting of the Stratosphere Committee of the Academy of Sciences of the USSR.

The results of the measurements are represented in the figure. The curves represent the results of 12 day measurements. On the left the logarithms of the energy of scattered light in relative units are plotted on the axis of ordinates; thus, from the curves in the figure it is possible to obtain for any given z the distribution of energy in the spectrum of scattered sky-light in relative units. Dotted lines show the Umkehr-Effekt curves, i. e. changes with z of the logarithm of the ratios: I_{3110}/I_{3260}, I_{3050}/I_{3260} and I_{2980}/I_{3260}.



The curves of the figure show that the Umkehr - Effekt takes place for all wave-lengths up to 2980 Å incl. The minimums of the curves for lesser wave-lengths are shifted in the direction of smaller values of z.

Two steps have been observed on the curves for $\lambda = 2980$ Å (and accordingly, there are two minimums on the Umkehr-Effekt curves for this λ), which in respect of their position relatively to axis z, as well as with regard to their magnitude, are well reproducible from day to day and apparently point to the existence of a second light absorbing layer, $\lambda = 2980$ Å (the Kennedy-Heaviside layer?).

The results show that measurements conducted with a counter, due to the great sensitivity of the apparatus, provide a much ampler material than the methods used heretofore (the Umkehr-Effekt observed by us for small λ) and therefore permit of making more exact calculations of ozone distribution. The corresponding computations will appear in print shortly.

In conclusion we want to point out that there exists a considerable similarity between the Umker-Effekt curves obtained and the curves for anomalous morning course in direct sunlight as described in the preceding article; this similarity and the close coincidence of the minimums of these and the other curves (a certain discrepancy may be caused by the fact that somewhat different wave-lengths were measured in both cases) makes us assume a somewhat more general mechanism governing both effects, which does not seem to be fully accounted for by the Umkehr-Effekt of Götz. It would seem that the latter needs to be somewhat reconsidered and supplemented in certain of its aspects.

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BIOPHYSIQUE

RECHERCHES SUR L'ADAPTATION VISUELLE PÉRIPHÉRIQUE AU COURS DE 24 HEURES

Par A. A. NOVITSKIJ

(Présenté par P. P. Lasareff, de l'Académie, le 27. X. 1937)

1. Les recherches de P. P. Lasareff sur la théorie ionique de l'excitation ont montré que l'accroissement de la sensibilité périphérique de l'œil dans l'obscurité après l'éclairage préalable dépend de la restitution du pourpre visuel dans les bâtonnets de la rétine (1). D'après l'hypothèse admise par Lasareff dans son premier travail (1914) (2) la restitution du pourpre est une réaction chimique monomoléculaire. La comparaison des données expérimentales (Nagel, collaborateurs de Lasareff) avec la théorie a démontré que la réaction du premier ordre représente assez bien l'allure de la sensibilité mesurée dans les expériences (3).

Les travaux de S. Hecht (1920) (4), se basant sur les idées de Lasareff, ont affirmé que la réaction de la restitution du pourpre est une réaction bimoléculaire. En répondant à cette affirmation de Hecht Lasareff (5) a démontré que la sensibilité mesurée dans l'obscurité pendant un temps assez court (une heure environ) par Nagel, Lasareff et Hecht ne permet pas de résoudre la question de l'ordre de la réaction et la courbe de la sensibilité E coïncide dans cet intervalle de temps avec les courbes de la réaction du premier, du second et même du troisième ordre.

Pour la résolution du problème de l'ordre de la réaction dans la rétine au cours de l'adaptation, on doit, comme l'a montré Lasareff, observer la sensibilité E de l'œil pendant 6, 12 et même 24 heures. C'est la cause des recherches entreprises, d'après les indications de Lasareff, par N. I. Popov et N. A. Popov (6,7).

L'adaptation était mesurée au cours de ces recherches pendant 6 et 12 heures. Les résultats n'étaient pas définitifs; mais la réaction du premier ordre exprimait les données expérimentales mieux que celle du second ordre.

Plus tard, d'après l'indication de Lasareff, A. S. Achmatov (8) a entrepris les recherches de l'adaptation pendant 24 heures. Pendant cette étude Achmatov observait beaucoup de fois la courbe de l'adaptation de son œil. Ces expériences ont démontré que pendant 6 et parfois même 8 heures on observe une coïncidence des données expérimentales avec la courbe théorique, admettant la réaction du premier ordre. Les effets importants découverts par Achmatov étaient les sauts brusques de la sensibilité observés au cours de l'adaptation. Ces sauts se produisaient aux moments différents qui n'étaient pas bien définis et qui changeaient chaque jour. D'après Lasareff les sauts découverts par Achmatov doivent dépendre des processus se produisant dans les centres de l'écorce cérébrale. Il était donc intéressant d'étudier l'adaptation non sur une personne mais sur une série de personnes dont les

centres peuvent fonctionner d'une façon différente. C'est la cause pourquoi Lasareff m'a proposé d'entreprendre une étude de l'adaptation visuelle périphérique au cours de 24 heures chez une série de personnes.

2. La sensibilité E pendant l'adaptation visuelle périphérique était mesurée avec l'adaptomètre de Nagel. La valeur de E était obtenue ou sans l'adaptation préalable ou avec l'adaptation préalable, permettant de trouver les valeurs de E correspondant à l'adaptation secondaire (9).

Les manipulations sur l'adaptomètre ont été exécutées toujours par le même expérimentateur (Novitskij) qui faisait dans l'appareil tous les mouvements nécessaires pour obtenir la limite de l'excitation de l'œil d'une personne étudiée. Toutes les expériences de l'adaptation ont été exécutées avec la vision binoculaire. Les mesures de la sensibilité E étaient faites chez 5 personnes dont l'âge variait entre 20 et 26 ans. A cet âge correspond le maximum de la sensibilité, ce qui permet de comparer immédia-

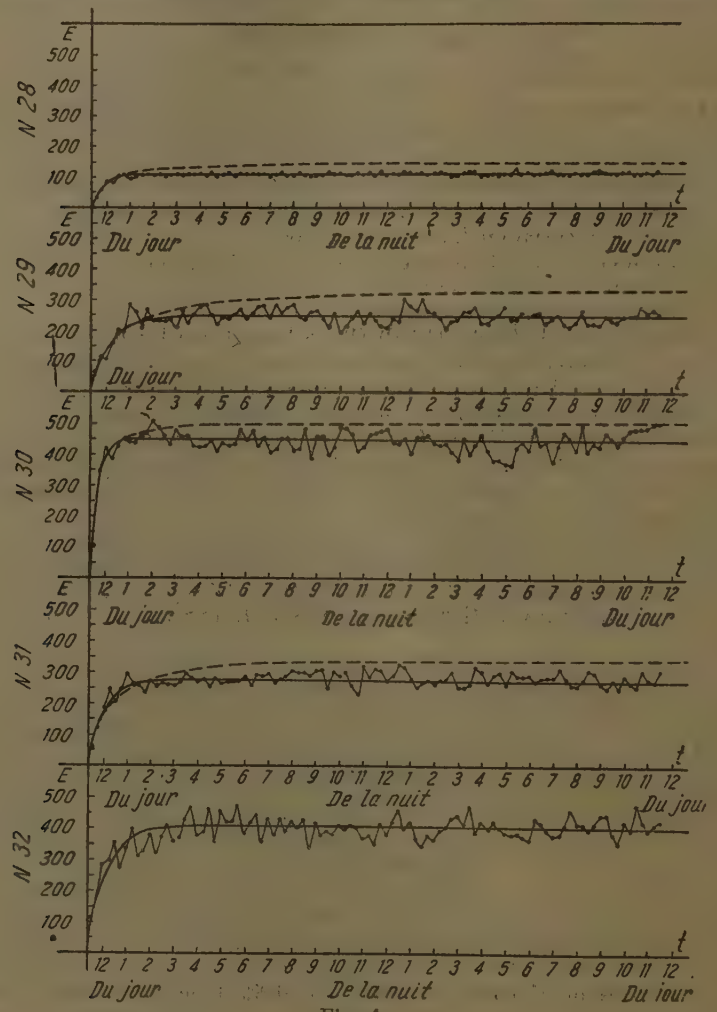

Fig. 1.

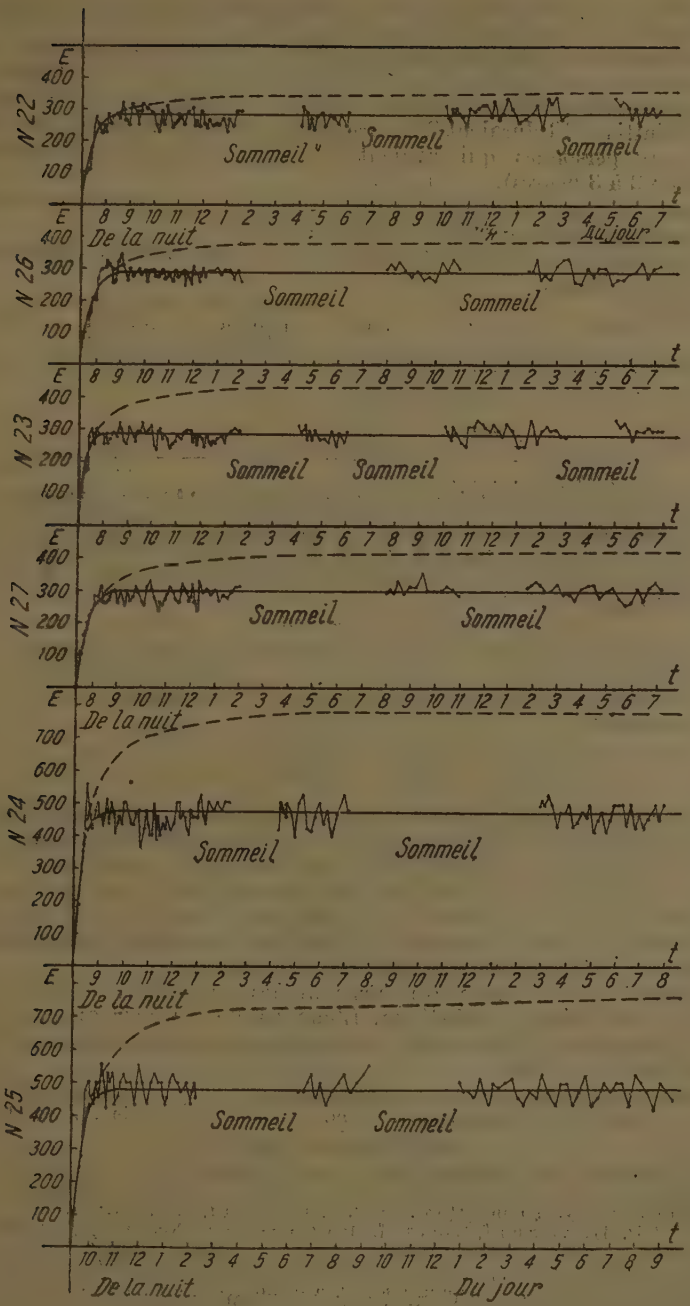

Fig. 2.

tement les mesures exécutées chez les personnes étudiées. Une personne avait l'âge de 20 ans, une—22 ans, deux—23 ans et une 26 ans.

Les recherches ont été exécutées entre le 29 mai et le 30 septembre (1937). Le nombre des expériences était 26; parmi ces expériences 7 doivent être envisagées comme préalables, durant 6, 8 et 16 heures. Parmi les 19 expériences définitives 13 duraient 24 heures sans sommeil. 6 expériences correspondent aux personnes qui avaient le sommeil pendant l'adaptation (sommeil de 2 à 3 heures).

D'après ces expériences on calculait l'allure de la courbe de la sensibilité E immédiatement après l'éclairage préalable.

Le nombre des expériences exécutées avec les diverses personnes est donné sur le tableau.

Nom	Age	Avec adaptation préalable	Sans adaptation préalable	Total
1. Sergienko	23	—	3	3
2. Savkov	23	2	3	5
3. Sanina	20	2	2	4
4. Kourapov	26	2	3	5
5. Bagarov	22	—	2	2
Total		6	13	19

3. Les résultats des recherches sont reproduits dans les deux figures (fig. 1 et fig. 2). Les courbes, reproduites sur la fig. 1, donnent le changement de E au cours de 24 heures sans sommeil. E est une moyenne de deux valeurs observées de E. Sur la fig. 1 le nombre de la courbe N correspond à la personne étudiée suivante:

N	28	29	30	31	32
Personne étudiée	Sergienko	Savkov	Sanina	Kourapov	Bagarov

Sur la fig. 2 le nombre de la courbe N correspond à la personne étudiée suivante:

N	22 26	23 27	24 25
Personne étudiée	Savkov	Kourapov	Sanina.

4. Les résultats obtenus pendant mes expériences permettent de les comparer avec la théorie de l'adaptation de Lasareff et celle de Hecht.

Sur les figures (fig. 1 et fig. 2) la ligne continue représente l'allure de E d'après l'hypothèse de la réaction monomoléculaire. La courbe formée des lignes courtes donne la valeur E calculée avec l'admission de la réaction bimoléculaire. On peut voir que ce n'est que la réaction monomoléculaire qui coïncide bien avec les résultats des expériences.

Les courbes de la sensibilité montrent les variations, les fluctuations ayant la forme presque périodique. L'analyse de ces courbes est exécutée sous la direction de P. P. Pavlov. Les résultats de cette analyse seront décrits dans un article spécial.

Institut A. M. Gorky de Médecine expérimentale.
Section de Biophysique. Moscou.

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CHEMISTRY

UNLIMITED SOLUBILITY OF METALS IN THE SOLID STATE

By N. AGEEV

(Communicated by N. S. Kurnakov, Member of the Academy, 28. II. 1938)

When a solid solution is formed, the lattice of the solvent contains atoms of the dissolved component. Both kinds of atoms form a common crystal lattice. If a foreign atom is introduced, the periodicity of the force field of the lattice becomes distorted. This distortion is largely due to the increase in the number of electrons when a metal with a greater number of valency electrons than the solvent is introduced into the solid solution. An electron may occupy a number of definite energy states in the crystal lattice. As the number of electrons gradually increases, they fill up the lowest energy zone. When this so-called Brillouin's first zone is filled, the next electron can be situated only in the next energy zone, i. e. it must have much more energy. If the first zone is absolutely filled, the crystal which forms should be an insulator provided the curve showing the energetic state is discontinued at the boundary between the first and second zones. The crystal will conduct electricity only if the electron can continuously pass into a position of greater energy. This condition is fulfilled in the case of metals since they conduct electricity. If there is no pronounced discontinuity of the energy change, the crystal will be a semi-conductor.

A continuous change of the energy state will be observed in metals, only if the energy of the electron is considered without reference to the direction of its motion. If, however, the transfer of the electron's energy is taken in a definite direction, there will occur discontinuity zones in the metal as well. Such energy jumps, which occur in the metallic crystal when the energy of the electron is transferred in a definite direction, can be investigated by using the notions of wave mechanics.

An electron moving with definite velocity in the crystal lattice is connected with the de-Broglie wave, the length of which is $\lambda = h/m\upsilon$. If the final volume of the crystal lattice of the metal is taken, the de-Broglie waves formed will be standing waves with a wave-length depending on the volume of metal chosen. If Pauli's principle is taken into account, the maximum energy of the electron and the minimum wave-length corresponding to it ($\lambda_{min} = 2(\pi V/3N)^{1/3}$, where V is the volume of metal and N, the number of electrons in it) may be calculated. Consequently the lattice contains a set of wave-lengths down to a certain minimum which decreases as the number of electrons in a given volume of metal increases. By increasing the number of electrons, by introducing a metal of higher valency into the solid solution, the wave-length can be reduced to the limit, where in case of its normal

incidence on some definite atomic surfaces in the crystal lattice it falls into the sphere of Bragg's condition of reflection: $\lambda = 2d \sin \theta$. These are mainly the surfaces with the greatest intersurface distance d. If the energy of an electron, moving normally to a series of atomic surfaces, is plotted in inverse relation to the wave-length, the result is a curve discontinued at a definite wave-length. This wave-length λ_c is the critical one determined by Bragg's law, and if an electron with such a wave-length entered the crystal from outside, it would undergo full interior reflection. Such electrons cannot exist in the metallic crystal.

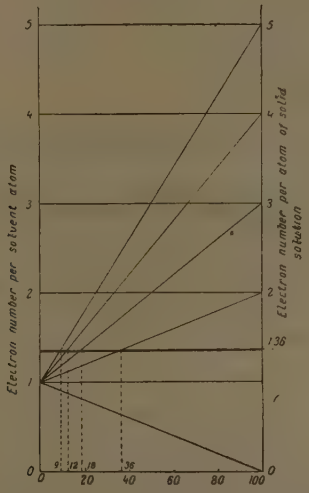
These assertions give grounds to the Jones' theory, according to which a given crystallic structure can exist only if the electrons are able to arrange themselves according to their energy along a branch of the energy curve within the limits from $1/\lambda = 0$ to $1/\lambda = \frac{1}{2}d \sin \theta$. If their energy is increased above these limits, the structure becomes unstable and has to pass into another one which will make the above relation more favourable.

Let us apply the Jones' theory to lattice of a cube with centred faces. As the structural factor shows, the reflections in this lattice should proceed primarily from planes $\{111\}$. The critical wave-length here is $\lambda_c = 2a/\sqrt{3}$, where a is the cell parameter. The cell of a cube with centred faces contains four atoms and therefore $a = (4\ V/N_A)^{1/3}$, where V is the volume of metal and N_A, the number of atoms in this volume. The critical length may therefore be expressed as $\lambda_c = 2(4V/N_A)^{1/3}\sqrt{3}$. Equating the minimal wave-length, expressed in relation to the number of electrons, to the critical wave-length, we get: $N/N_A = \pi\sqrt{3}/4 = 1.360$ after simplifying.

The Jones' theory shows that the concentration of valency electrons has a great influence over the ability of metals to form solid solutions. The ratio of the number of electrons to one atom for a metal with the lattice of a cube with centred faces may reach a maximum value of 1.360.

Let us see under what conditions the formation of a continuous series of solid solutions is possible according to Jones' theory. The influence of the electronic concentration on the ability of metals to form solid solutions is shown graphically in the figure. If we assume that the solvent has one valency electron per atom and if the atoms of the solute have a zero valency, then a complete replacement of all the atoms of the solvent will reduce the electronic concentration from unity to zero. If the solvent has the lattice of a cube with centred faces, the critical electronic concentration (1.360) cannot be reached and, consequently, the formation of continuous series of solid solutions is possible. By introducing atoms with a single valency electron into the same solvent, it is possible to keep the electronic concentration constant while the composition of the solid solution changes from 0 to 100 atomic per cent. If a bivalent metal is dissolved in the same univalent solvent, it is possible to produce an electronic concentration of two, when the atoms are completely replaced. Consequently the critical concentration is reached before the atoms in the lattice are completely replaced and it is impossible to produce a continuous solid solution under these conditions. As we see in the figure, the straight line expressing the change in electronic concentration, when a bivalent metal is dissolved in a univalent one, intersects the horizontal of the critical electronic concentration of a cube with centred faces (1.360) in a point corresponding to 36 atomic per cent of the dissolved metal; this is the saturation limit for this solid solution. If the number of valency electrons in the solute atoms is increased, the conditions become even less favourable for the formation of a continuous solid solution, and therefore only a limited solubility is possible and the saturation limit becomes still less, as we see in the diagram in the figure.

When the number of valency electrons of both of the metals in the solution is equal, the electronic concetration stays constant, just as we saw above in the case of two univalent metals. This case, however, is of much greater importance and may be expanded to metals of higher valency, since in all these cases the constant electronic concentration involves a constant quantity of the Brillouin's zone to be filled, i. e. such as did not change with altering composition and makes thus fully possible to obtain a continuous solid solution.



Atomic per cent of solving metal.

The following rule may be given: it is possible to produce a continuo u series of solid solutions of two metals, if the electronic concentration is below the critical, or is costant within the limits where the solid solution is homogeneous. It is necessary to fulfill this condition in order to produce a continuous series of solid solutions, but this condition is not enough that we may draw conclusions as to the nature of the interaction between the metals. There are many metals which do not form continuous solid solutions although the condition above was accurately fulfilled. This is due to the fact that although the electronic concentration is one of the factors, it is not the only one.

The following systems are examples which confirm this relation between the formation of a series of solid solutions and the electronic concentration: Cu-Au, Ag-Au, As-Sb, Sb-Bi, Mo-W, Cu-Ni, Au-Pd, Cu-Pt, Au-Ni, Co-Ni, etc., in which the metals Co, Ni, Pt, Pd are considered to have a valency of zero, Au, Cu and Ag are univalent, As, Sb, Bi—pentivalent, Mo and W are in the same subgroup, but are anomalous in the manner in which their electronic levels are filled.

The identity of the crystal structure of metals which form continuous solid solutions necessarily follows from the influence of the electronic concentration and, consequently, from the relation between the ability to form unlimited solid solutions and the periodic system of elements. As we know, metals of the same subgroup in the periodic system, i. e. those which have the same number of valency electrons, have also the same crystalline structure. Thus, among the examples given above, metals of the transitional

group and gold, silver and copper have the lattice of a cube with centred faces; arsenic, antimony and bismuth, a rhombohedric lattice; molybdenum and tungsten, a centred cube. From the observation that only metals which have similar crystal lattices form continuous series of solid solutions, one drew the conclusion that analogy in crystalline structure is a necessary condition for such an interaction between metals. It may be assumed that this condition is only a consequence of the necessity to keep the electronic concentration constant or below the critical point, when an unlimited solid solution is formed.

Institute of General and Inorganic
Chemistry of the Academy of
Sciences of the USSR.

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GEOLOGY

ON THE DOMES OF THE REGION ADJOINING THE DZHEZKAZGAN

By B. A. PETRUSHEVSKY

(Communicated by A. D. Arkhangelsky, Member of the Academy, 10. III. 1938)

In the autumn of 1937 the author discovered three domes of Paleozoic rocks in the northern part of the Chui-Sarysui depression, among horizontal Tertiary deposits; the central portions of two of these domes were found to be composed of gypsums.

The domes are located in the plain adjoining the righthand bank of the middle course of the Sary-su river, 30—35 km southwards from the border of the Paleozoic Kazakh country. Dzhezkazgan lies 120 km N.-N.-E. of the domes. The plain is composed of Eocene—Lower Oligocene marine clays, and of younger continental Oligocene rocks, forming numerous table monadnocks, clearly showing that the beds are lying horizontally everywhere. One of these terraces, Beleuty-Nura, is located 700—1000 m from the extreme northern dome.

This dome, Rakhmet-nura, is about 1000 m in diameter, from these 450—500 m fall to the lower central gypsum zone; the elevated marginal portions are composed of limestones; their elevation above the surrounding steppe is 40—60 m, above the gypsum core—10—15 m. In the plan the dome presents a rather regular circle. The strike of the beds in the calcareous external ring also forme a circle, but a less regular one, with numerous local disturbances. The dip is everywhere directed downwards from the center of the dome; angles, from 30 to 90°, most frequently 40—50°; almost no overturn is observed. The gypsums of the central part are crumpled to a considerable extent, no regularity being observed in the attitude of the beds.

No contact may be seen between the gypsums and limestones, but the tectonic nature is beyond doubt. The gypsums are chiefly cryptocrystalline, whitish-grey, weathered at the surface to a considerable extent; secondary gypsum in large plates is frequently encountered. Grey limestones, compact with a large number of Visean fossils: *Echinoconchus elegans* Wel., *Productus longispinus* Sow., *Pr. sulcatus* Sow., *Pr.* cf. *pugilis* M. Wood, *Gigantella rectestrius* Gröb., *Cyanthophyllum jagovkini* Gorsky, *Litostration portlacki* Edw. and Heim, and others. Their thickness is about 100 m. The high external ring of the dome, formed by these limestones, is broken in about 250 m along the N.-E. border, where the gypsums are directly overlain by meso-(?) cenozoic deposits.

Here, 700—800 m from the Oligocene escarpment Beleuty-nura, 15—20 m above its edge, pebbles have been found, very similar to those encountered in the Cretaceous rocks of the region, fragments of glauconite Paleocene sand-

sandstones and teeth of sharks. In a normal section the Paleocene rocks must lie 40—50 m below the surface of the steppe, i. e. 100 m below the mark, where they have been found. Lower along the slope marine Paleogene (Eocene-Oligocene) clays are exposed, still lower Oligocene continental clays.

The dome Kock-tyube is located 6—7 km S.-S.-W. up to 3 km along the long, and up to 2 km along the short axis, raised 80—90 m above the steppe. It presents a short asymmetric fold with a full south-eastern limb and a very much crumpled and reduced north-western limb. The southern portion of the dome is covered with turf. In the northern portions the normal N.-E. strike is seen in places to pass into the normal strike, almost a latitudinal strike. The full limb is dipping 35—50° on an average; in the reduced limb the dip is steeper, up to 90°, and overturned.

The oldest rocks of the dome are reddish clays, which may be referred to the transitional Turnaisian and Visean beds from their analogy to the sections of northern regions (Karsakpay). This cannot be upheld, however, owing to the absence of fauna. Higher up grey compact limestones are lying, entirely analogous to those described at the Rakhmet-nura and with a similar fauna, up to 100 m in thickness. They are overlain by 260—280 m thick pack of reddish and greenish sandstones, the so-called «Dzhezkazgan» series of the northern regions, where it is up to 600 m thick. The youngest Paleozoic rocks of Kock-tyube are red sandstones of the overlying «redcoloured» series which is only 30 m thick against the 300 m of the Dzhezkazgan series.

Along the N.-W. border of the dome it is seen that the Visean limestones (at a height of 30—35 m above the steppe) are overlain by fragments of Cretaceous (?) deposits (pebbles and wood), and below these along the slope a basal Paleocene conglomerate with oysters and teeth of sharks is seen, inclined at an angle of 5—7°, somewhat farther from the dome Paleocene quartzite, Eocene-Oligocene clay, and finally, continental Oligocene clay bassets may be observed. Thus, the Paleocene rocks are lying here also 70—80 m above the normal level.

The third dome, Bureinak, is located 2—3 km E.-S.-E. of Kock-tyube. It presents in the plan an irregular ellipse, about 1000 m along the long, and about 650 m along the short axis, up to 37 m above the surrounding steppe. In contrast with the above domes Bureinak is composed merely of gypsums, more frequently cryptocrystalline, but sometimes also secondary large plates are encountered. The gypsums are crumpled to a considerable extent and contain fragments of Lower-Carboniferous limestones; along the margins of the dome the gypsums are usually overturned, but their strike strictly follows the outlines of the dome. Only along the N.-E. border the red coloured sandstones of the Dzhezkazgan series, not over 25—30 m in thickness, and fragments of the Cretaceous escaped destruction; Visean limestones with *Gigantellae* are absent.

The above said enables several conclusions to be made. First of all the tectonic contact between the Visean gypsums and limestones is beyond all doubt, since in Kock-tyube the limestones are not underlain by gypsum, and at the Bureinak the gypsum is overlain by younger rocks.

The difference of the stratigraphical composition of the domes located side by side, the absence of whole series (Visean limestones at the Bureinak, Dzhezkazgan, sandstones at Rakhmet-nura) and their thinning out, as well as the difference of the section of Kock-tyube (the most complete of all the three) from that of Dzhezkazgan-Karsakpai, all of it makes it possible to state that the dislocations have been very intensive here, though at first sight it is hardly noticeable owing to insufficient exposure.

The formation of the domes in their present aspect should be referred to the Alpian epoch of folding, subsequent to the Lower Oligocene, since the deposits of this age are disturbed.

It should be emphasized that the domes have not to be regarded as due to the Alpian movements of the whole region. These movements did not produce here any clearly pronounced folded structures: the Cretaceous and Tertiary deposits of the Turgai-Chui steppe stretch practically in horizontal direction; in the S.-W. portion of the Kazakh Paleozoic country gentle uplifts took place, only up to some tens or one—two hundreds of meters. The intensively dislocated domes should not be considered therefore as normal Paleozoic folds with gypsum cores.

Doubtless the causes of their formation should be sought in other phenomena, most probably in the penetration of fluid rocks, i. e. the domes should be regarded as diapir folds; at present it is indistinct, however, whether their cores of penetration are only gypsums or salt will be also found underneath, no traces of which have been found as yet at the domes (neither in the sections nor in the form of salt springs, etc.). This has to be decided in the course of further studies; so far it is only possible to state the very fact of penetration. The depth, from which the gypsums have been uplifted, is (by analogy with the northern regions) 2000—2500 m, considering from the base of the Visean deposits.

It is impossible so far to speak definitely about the age of the gypsums, since only some tens of kilometers northwards, in the Kazakh country, no gypsum strata of significant thickness* have been discovered below the Visean. Evidently we have here an instance of an extremely rapid and abrupt alteration of the facies. Turning to the character of the pre-Visean deposits of the western part of Kazakhstan, we see that both in Karsakpai and in Karatau, i. e. in the north and in the south the Devonian is presented by continental red-coloured rocks. It is highly probable that the gypsums of the domes are one of the facies of these continental beds.

The alteration of Devonian facies southwards from Ulu-tau is attested by the fact that the marine sediments, developed in the north in the uppermost part of the Devonian, southward are very soon entirely replaced by continental deposits.

Considering the rapidity and abruptness of the alteration of facies, the assumption suggests itself that salt appears synchronously with gypsums being, moreover, the cause to which the domes under investigation owe their existence.

Geological Institute
of the Academy of Sciences of the USSR;

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* A bed of alternating gypsums and limestones, up to 200 mg in thickness, has been discovered recently by boring much farther north of our region, 90 km westwards from the middle course of the Ishim river; the bed has not been bored to the base; the age of the gypsums is still undefined, they may prove Upper Paleozoic and Middle and Lower Carboniferous and even Devonian.

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PETROGRAPHY

FIRST NEPHELINE SYENITE IN TRANSCAUCASIA

By D. S. BELIANKIN, Corresponding Member of the Academy, V. P. PETROV and R. P. PETROV

Up to the present time not only nepheline syenite, but nepheline rocks in general were not known with full certainty in Transcaucasia. Though A. S. Ginsberg in his summarized report, concerning the petrography of Soviet Armenia, mentions several descriptions by different authors of Armenian nepheline bearing effusives, he immediately justifies that these descriptions must be considered with caution since they are not accompanied by optical and chemical data: «and presently we may suppose either an error in definition or assume that the samples have been mingled with some other collections» (1934, p. 58). And really, even in the most certain, apparently, case reported by Ginsberg—nepheline bearing porphyrite near Jarkhetch described by J. J. Tanatar, nepheline is mentioned among the components of the ground mass «consisting of the decomposed plagioclase microclines, water-transparent grains of nepheline, chlorite, and calcite». Nepheline is hardly possible in the paragenesis of such a kind.

A. P. Gerasimov's indication concerning nepheline, discovered by this author in teschenites on the Iora river in Kakhetia, is of greater importance. But though the author gives a very comprehensive description of teschenites, his details concerning nepheline are but qualitative ones, hardly sufficient for the full identification of this mineral.

Contrary to the just mentioned and similar doubtful nepheline rocks we observed in the summer of 1936 a real nepheline syenite, though up to the present time only in form of the river boulders found in the beds of the left affluents of the Miskhana river in North Armenia. On the geological map (a copy from the geological map by V. N. Kotlar, found in his work «Materials Concerning the Study of Ore Deposits in Northern Part of Armenian S. S. R.», 1934) the corresponding localities of these discoveries are marked by numbers 126 and 127. Judging by their position, the location of country nepheline syenites must be looked for in the region of the syenite massif of mountain Ker-Ogly, marked on Kotlar's map.

The samples of nepheline syenite from the point 126 may be described as follows. The rock is coarsely crystalline and relatively leucocratic one with prevalence of anorthoclase and with subordinate nepheline. The quantity of plagioclase is small. Coloured minerals pyroxene and amphibole also make only single per cents.

Anorthoclase occurs with its usual deviations from orthoclasticity: P (001)—N_g 86°, N_m 8.5°, N_p 82.5°; M (010)—N_g 7°, N_m 83°, N_p 90°; $P:M$ =91°; $2V$ =—61,—64°. It is somewhat clouded owing to kaolinization and with

a rather considerable perthitization. Refraction in fresh and not perthitized portions: $N_g' = 1.522 \pm 0.003$, $N_p' = 1.518 \pm 0.003$, which corresponds to the approximate content in them of 15—20% of albite molecules. Chemical analysis of picked out anorthoclase was made by V. A. Egorov. The obtained values are given in Column 1 of Table 1. The computation of elementary components from these data gives the following values: Or=60.4%, Ab=29.8%, An=4.8%, Ce=0.62%. The residue of argillaceous substance 4.65%.

Therefore, the excess of chemical albite (40%) comparing to the optical one (20%) serves as a certain measure of perthitization of our mineral.

Table 1

	1	2	3	4	5	6	7
SiO_2	63.65	44.31	55.24	47.67	4.22	3.35	38.38
TiO_2	0.04	Absent	0.56	0.03	—	0.53	6.07
Al_2O_3	19.63	33.56	20.77	16.78	3.15	0.89	10.19
Fe_2O_3	0.41	0.60	1.64	0.34	0.90	0.40	4.58
FeO	—	—	1.85	—	0.40	1.45	16.61
MnO	0.01	0.01	0.22	0.02	—	0.20	2.29
MgO	0.12	0.28	0.62	0.11	—	0.51	5.84
BaO	0.25	Absent	0.10	0.16	—	—	—
CaO	0.97	2.43	3.39	0.91	2.08	0.40	4.58
Na_2O	3.52	10.75	4.48	3.49	0.47	0.52	5.96
K_2O	10.20	6.42	7.97	7.57	—	0.40	4.58
H_2O_{-100}	—	—	0.54	—	1.13	—	—
Loss at heating	1.31	1.25	1.68	1.01		0.08	0.92
P_2O_5	—	—	0.11	—	0.11	—	—
CO_2	—	—	0.47	—	0.47	—	—
S	—	—	0.04	—	—	—	—
Cl	—	—	0.02	—	—	—	—
Total	100.11	99.60	99.70	—	—	8.73	100.00

1. Composition of anorthoclase.
2. Composition of nepheline.
3. Total composition of rock.
4. Oxides forming 67.8% of anorthoclase and 10.2% of nepheline. The composition of minerals is the same as in columns 1 and 2 of Table 1; their percentage see in column 9 of Table 2.
5. Oxides forming 1.3 of plagioclase № 50, 1.3% of magnetite, 9% of thomsonite with formula: $3CaO.Na_2O4Al_2O_3.\ 8\ SiO_2.10\ H_2O$, calcite and apatite (according to the whole quantity of CO_2 and P_2O_5 in the rock).
6. Residue accounting for the mixtures of pyroxenes and amphibole.
7. The same as in 6 recalculated for 100.

Plagioclase occurs in form of relatively sparse idiomorphic crystals of zonal structure. In their cores the content of An reaches in the limit 70% (extinction $\perp PM + 41°$); then follows quantitatively prevailing part with 48% An (extinction $\perp P$ 26—27°); in the narrow marginal zone oligoclase is about № 20 (extinction $\perp PM$ 1—2°).

Nepheline with $N_g = 1.539 \pm 0.003$ and $N_p = 1.535 \pm 0.003$. It is very often replaced by the aggregate zeolite: the latter by its optics approaches the nearest to thomsonite (direct extinction, negative optical sign, high birefringence, $N_g' = 1.527$, $N_p = 1.519$).

Chemical composition of the picked out freshest nepheline see in column 2 of Table 1.

The products of nepheline decomposition, except thomsonite, contain sericite, which is also partially present in the feldspathic part of the rock.

Pyroxene—monoclinic, belonging to augite series with the following optical constants: $CN_g = 44°$; $2V = +60°$; $N_g' = 1.732 \pm 0.004$; $N_p' = 1.700 \pm 0.004$. Due to its intense refraction it must be high in iron oxides.

Amphibole with the strongest absorption and the following pleochroism colours: N_g—dark-green to black, N_m—dark-green, N_p—brown. Extinction $CN_g = 32—33°$; $-2V = 18°$; $N_g' = 1.708$; $N_p' = 1.696 \pm 0.004$. These constants correspond approximately to hastingsite, but the absence of blue tint along N_g is characteristic (small quantity of alkalies?).

Scheme of nepheline-syenite pebblestone deposits (Geological map of Kotlar).
1—Tertiary diorit intrusions; *2*—Tertiary syenite intrusions; *3*—Eocene porphyrite series; *4*—Pre-turonian intrusions and crystalline shists.

Thin sections show magnetite and apatite as accessory minerals. The artificial heavy residues allow also to join to the latter rutile, zircon and titanite, the latter mineral being also macroscopically discerned in form of sparse grains in the blocks of nepheline syenite.

The quantitative computation of minerals in thin sections enabled us to obtain the volume per cent given in columns 1—5 of Table 2.

Table 2

						Average	Accounted for crumbling out of pyroxene and amphibole	Assumed specific weight	Weight %
	1	2	3	4	5	6	7	8	9
Potassium feldspar	66.7	83.0	70.5	71.2	73.9	72.4	69.1	2.53	67.8
Nepheline	13.5	3.3	14.2	16.5	3.2	10.7	10.2	2.58	10.2
Thomsonite	15.5	5.6	13.0	6.7	10.1	10.4	9.9	2.35	9.0
Sericite	2.7	2.5	1.1	1.4	2.9	2.1	2.0	2.76	2.2
Plagioclase	0.1	2.5	—	0.5	4.3	1.4	1.3	2.67	1.3
Amphibole	0.3	2.0	0.7	1.1	2.4	1.2	3.7	3.1	4.4
Pyroxene	0.5	0.8	—	1.3	2.4	1.0	3.1	3.2	3.8
Magnetite	0.7	0.3	0.5	1.3	0.8	0.8	0.7	4.9	1.3
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	—	100.0

Owing to the relative coarseness of the rock granulation we observe very significant deviations between the separate preparations. Besides, on account of the crumbling out of dark-coloured minerals in the process of polishing their quantity in the microscopical calculations is certainly diminished as compared to the actual one. We have obtained a more reliable value 7.6% by means of a special macroscopical calculation on the surface of our samples.

The average calculation with such a correction is given in column 7 of Table 2. The column 9 contains the weight per cents.

In addition to just mentioned calculations we made also the total chemical analysis of our rock. The data of this analysis made by V. A. Egorov are given in column 3 of Table 1.

Columns 4, 5 and 6 of Table 1 contain the results of recalculation of analysis for minerals. The last column (7) contains the approximate composition of the sum of metasilicates—pyroxene and amphibole. These metasilicates contain, in agreement with their colour, much of TiO_2, FeO and alkalies, though the quantity of these oxides due to the peculiarities of calculation, is, of course, very much exaggerated.

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GENETICS

GENES *SCUTE* AND *ACHAETE* IN *DROSOPHILA MELANOGASTER* AND A HYPOTHESIS OF GENE DIVERGENCY

By A. S. SEREBROWSKY, Corresponding Member of the Academy

In the present communication are treated the genes *scute* and *achaete* or parts of a single gene *achaete-scute*. They are located near the left end of the *X*-chromosome of *Drosophila melanogaster*, in genetic terms at the zero morganid, cytologically—in the second disk (Meller and Prokofieva, 1934), more precise, 1B23. The two normal allelomorphs (ac^+ and sc^+) of these genes are necessary for the normal development of the bristles, viz. of the majority of the macro- and microchaetae which cover every part of the body of the fly. Recessive mutations of both *achaete* (*ac*) and *scute* (*sc*) are followed by the disappearance of certain bristles. Maximum denudation of the fly occurs when ac^+ and sc^+ are totally removed owing to deficiencies ($\overline{ac}$ and $\overline{sc}$); this may be actually produced—and is particularly impressing—by means of crossing-over between two differently inverted chromosomes. The left end of the chromosome y^{3P} in joining the right end of the chromosome sc^8 gives $\overline{ac}$ (Meller, 1935), while the left end of the chromosome sc^8 with the right end of the chromosome sc^4 gives $\overline{sc}$ (obtained independently by us and by Sturtevant and Beadle).

Considerable literature is devoted to systematic study of *achaete-scute*; studies first began in our laboratory in 1928, and later on were started in other laboratories. We have recently re-investigated about 30 allelomorphs of *scute* and *achaete*; our studies involved a considerably longer number of bristles controlled by these allelomorphs. In the present communication we wish to throw light upon some interesting results of these studies.

Upon the publication in 1929—1930 of the scheme of structure of the basigene *achaete-scute* developed in our laboratory (Dubinin, 1929, 1930, and others), a number of opponents tried to prove that *achaete* and *scute* were mere qualitative alterations of one and same locus (Goldschmidt, 1931, Sturtevant and Schultz, 1931, Child, 1935, 1936). Soon it could be proved, however (Serebrowsky and Kamshilow, unpublished, 1931, Patterson and Stone, 1935), that, when mutation sc^8 took place, the left break of inversion has torn the basigene into two parts, leaving *achaete* in the left end of the chromosome and transporting *scute* to the right end. This was the crucial evidence for the spatial independence of the two parts. Then some of the geneticists have changed their view, assuming that *achaete* and *scute* were simply independent genes which happened, more or less by chance, to lie side by side.

The inadequacy of this view is proved by two facts. First, by the fact that the action of *achaete* is strikingly similar to that of *scute*, and, secondly,

by the fact that at the same time they are remarkably different; the latter circumstance apparently unveils the riddle of the evolution of these genes.

The resemblance between *achaete* and *scute* consists in the following. The two influence the bristles; the two normal allelomorphs stimulate the development of bristles; both influence different bristles. The two influence both macro- and microchaetae; the two produce various non-repeated allelomorphs; both have allelomorphs which cannot be arranged into series but instead appear to be step-like. The recessive allelomorphs of the two genes not only denude the flies but frequently produce a smaller plus-effect; both influence the development of the bristles without noticeably altering their size, without making them thinner, curved, or without leaving in their place basal rings. None of them produce any noticeable secondary effects, for example, anastomoses of veins, spreading wide the wings, etc., except cases of inversions and translocations, when the secondary effects may be connected with other loci.

Elements of this similarity are so conspicuous that it may be said without hesitation that *achaete* is more similar to *scute* than to any other gene among hundreds of them present in *Drosophila*; and *scute* is no less similar to *achaete*. Moreover, different step-allelomorphs belonging to *scute* may differ from each other not less than from allelomorphs of *achaete*. The allelomorphs sc^{11} ($=ac^1$), sc^5 and sc^6 or sc^{4sh} are different in that way, and one cannot tell which of them are the most similar, although sc^{11} is an allelomorph of *achaete*, and sc^5, sc^6 and sc^{4sh}—those of *scute*. Finally, *achaete* and *scute* may mutate together (sc^3, sc^{A1}, etc.) and may give together partial reversions to normal condition sc^{3-1}.

Facing this extreme unity of *achaete* and *scute* it is absolutely impossible to explain their association in the II disk by mere chance. If one consider that there are not less than 3000 disks in the chromosome set of *Drosophila* and that only a few genes are more or less similar to *achaete* and *scute*, the probability of the random side by side association of *achaete* and *scute* should be of the order of millionths of a unit. The probability becomes smaller still when one considers that there are serious grounds to admit that the gene *scute*-proper consists also not less than of two parts which are related to each other approximately in the same way as to *achaete*, i. e. which are capable of independent or joint mutation and of giving a phenotypically normal compound.

A valuable confirmation of the homology of *achaete* and *scute* we find in the following interesting unpublished observation by Prokofieva-Belgovsky. As was mentioned above, in the sc^8-inversion *achaete* is left at the left chromosome-end, while *scute* is transferred to the right. Prokofieva-Belgovsky has observed cases in salivary glands of this line where the chromosome formed a loop and conjugated with its ends, and in such a way, that one could clearly see the conjugation of the disks *achaete* and *scute* with each other. The accompanying drawing which was kindly put to our disposal by A. A. Prokofieva-Belgovsky for publication, clearly shows the situation described above. The assumption made by some geneticists, that the conjugation takes place here between «inert regions», does not conform to observation, since the inert regions are located to the right of *achaete*, and also to the right (or to the left, when bent) of *scute*, so that when these regions conjugate, the disks of *achaete* and *scute*, instead of coinciding, should be shifted with respect to each other.

Aside from the remarkable similarity between *achaete* and *scute*, they differ in a way which is no less remarkable; during a long time we could find no explanation for this difference. It consists in the fact that the bristles of the fly belong to two groups one of which is controlled by the gene *achaete*,

and the other—by the gene *scute*. Up to the present we have recorded over 50 different bristles or groups of bristles which may be classified into two groups almost without difficulties. This classification is presented in the following list. It is impossible to give here detailed description of all the bristles; we shall restrain ourselves, therefore, only to a list of designations, one part of which are widely used, the other applied in our laboratory (their description is being prepared for publication). This list clearly shows that almost every part of the fly's body bears bristles, which are controlled by *achaete* and *scute*; they are often mixed together in a surprising way, as for example on the fore-femur, on the back of the thorax (*dc*, *sa*), etc.

Location of the bristles	Bristles controlled by *achaete*	Bristles controlled by *scute*
Antennae	*ann*	*spi*
	ab	
	ac	
Upper and hind sides of the head	*fc*	*io* (?)
	io	*orb* 1, 2 and 3
	pro	*oc*
	ci	*po*
	v_1	v_2
	tz	v_1
Lower side of the head and the proboscis	*mig*	*g*
		vib
		mn (px, l, d)
Upper side of the thorax	*th*	*dca*
	dcp	*hs, hi*
	dca	*ps*
	sap	*na, np*
	misc	*saa, sap*
		paa, pap
		sca, scp
Thorax, sides and under part	*scx*	*ms*
	mp	*stpl a, st* and *p*
	stpl mi	
Legs (coxes and femur)	f_2, f_4	cx_1, cx_2, cx_3 *(m, sbm, sbl, l, srl)*
	fv	f_1, f_5
		fid
Wings	*vn*	*cdp cdd*
	cb	
Abdomen	*tg* (?)	*st*
		tg
		gt

The overwhelming majority of the bristles listed are strictly connected with one of the genes only and are wholly independent from or insignificantly connected with the other. Thus, the *interocellare* (*io*) are reduced to zero by many allelomorphs of *achaete* and are insignificantly affected by some allelomorphs of *scute*. But even flies with $\overline{sc}$ possess them. It is certain that *dorsocentrale ant.* (*dca*) and *verticale* 1 (v_1) depend on the two genes, but the effect of *scute* is much stronger. It is possible that a dual connection exists in the case of *femorale* 1 and 2, but one of the genes predominates. Below we shall speak of the interesting significance of these exceptions.

This remarkable distribution is the more puzzling that the genes (*hairless*, *fused*, etc.) that reduce the bristles in a qualitatively different manner, control also the bristles which are controlled by *scute* and *achaete*.

An explanation for this phenomenon we find in the assumption that the genes *scute* and *achaete* are closely related to each other, that they have

diverged having originated from a common ancestral gene. In connection with recent data on duplications which have been studied by us as well as by many others (Bridges, 1935, Kossikov, 1936, Meller and Prokofieva, Rappoport, Grüneberg, 1937, and others), the scheme of such an evolution may be interpreted in the following way. Let us suppose that there was a gene which controlled the bristles *a*, *b*, *c*, *d* and *e*. In case duplication

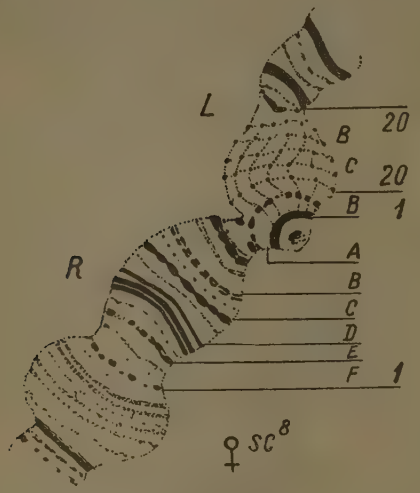

Fig. 1a.

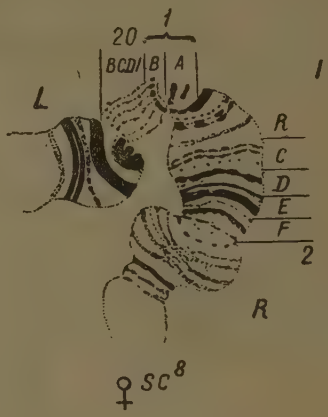

Fig. 1b.

Fig. 1, a and b. Cytological study of the *X*-chromosome sc^8 has shown that a proximal break in this chromosome passed between the disk 1B1 (which contains the loci *y* and *ac*) and the disks 1B23 (containing the locus *sc*), owing to which the disk 1B1 remained at the distal (left) end *L* of the chromosome, while the disks 1B23 proved to be transferred to the proximal (right) end *R*. Conjugation of the disjoined breaks of the disks 1B23 of the *X*-chromosome sc^8 may serve as evidence of the presence of homologous elements in these disks.

of this gene took place (for example, according to the scheme of the Bar-duplication), we shall have two genes $\overline{abcde}$ $\overline{abcde}$. Partial mutations taking then place in these genes should not be, as a rule, eliminated by natural selection, since in presence of duplication they will not sharply and harmfully disturb the phenotype, that is, produce denudation of the fly. Therefore, a mutation may be maintained, say, $\overline{a\ de}$ $\overline{abc\ de}$. After this in the second gene only such mutations could be maintained which affect *a*, *d* or *e*, but not *b* and *c*, since in the latter case the fly loses bristles *b* and *c*. Let the alteration $\overline{adc}$ $\overline{bce}$ survive. Two out of the five bristles, namely *a* and *d*, will be controlled only by the left gene, and two, *b* and *c*—only by the right one. The bristle *e* will still depend on the two genes, but upon separate mutation of the left or the right gene it will still exist (in case no partial alteration *e'* and *e''* takes place), so that it would appear to be independent from them. It is possible that such are the bristles of the tibiae, microchaetae near the vibrissae and some others, which do not disappear in *ac* and *sc*. Summing up, we should get the same picture of distribution of functions as is observed in the case of *achaete* and *scute*, since, under the protection of duplication, they may lose all the duplicate functions.

This principle of loss of duplicate functions by one of the homologues in the process of genic evolution is considered by us as an important (though not the single) explication of a great number of phenomena discovered by genetics. It should result in specialization of genes, when each of them fulfills only one function which is strictly limited and important for the life of the organism. Further mutation of such a gene would most often lead to lethal effect. What we call lethal genes are, therefore, the most ancient (counting from the moment of the last duplication), specialized and, consequently, the most irreplaceable genes. Their further evolution is possible mostly after a new duplication. Duplicate genes (typical mendelian ratio 15 : 1) would be, on the contrary, the youngest (counting from the moment of duplication, or, what is the same, from the moment of amphidiploidy), as in *Bursa bursa-pastoris* (Shull, 1929).

This principle clears up also the causes of the high autonomy of the genes as regards their influence upon the organism; it is manifested in the common fact that often with mutation of a certain gene only one strictly limited function is changed (for example, colouration of petals, the background of the wing, colouration of the eye), and that even the most careful investigations are capable of detecting only traces of pleiotropy.

Naturally such a process of evolutionary differentiation of genes is a very slow process. Comparative genetics of *Drosophila* has shown, however, that the gene *scute*, for example, is present in different and even in far distant species (*D. melanogaster*, *simulans*, *ananasae*, *virilis*, *pseudobscura A* and *B*, *hydei*, *willistoni*). Although we do not know the evolution of *Drosophila*, we may think, judging by the data on other groups of flies and by the fact that *Drosophila* is found in amber of the lower oligocene, that this differentiation has begun at least at the beginning of the tertiary, and even more probably, as early as in mesozoic. During this immense period there was surely enough time for hundreds of subsequent mutations in the genes *achaete—scute*, and there could arise the strict specialization which is observed now. We assume that the gene *scute* itself consists of several specialized parts of that sort which account for the apparent «disorderliness» of the localization of different centres, contained in its structure. However, the discussion of this problem does not fit in the plan of the present communication; we neither do discuss the question whether *achaete* and *scute* should be considered as different genes or merely differentiated parts of a single gene. Further study of *scute* and *achaete*, especially in other species of *Drosophila* and beyond the boundaries of this genus, should throw still more light on the most important genetic problem—that of evolution of the gene.

Laboratory of Genetics.
Moscow University.

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GENETICS

ON CHEMICO-GENETIC STUDY OF THE PLUM

By E. D. LEVINA

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The origin of the cultivated plum (*Prunus domestica*) was obscure until recently. First in 1930 Crane and Lawrence, on the basis of genetic and cytological investigations, have advanced a hypothesis in which the cultivated plum was considered as a product of hybridization between two wild species: *Prunus spinosa* and *P. cerasifera*. This hypothesis was experimentally verified by V. A. Rybin in the All-Union Institute of Plant Industry. This investigator has discovered in the region of the Maikop Experiment Station (North Caucasus) various forms of natural hybrids between the two wild species; with regard to a series of morphological characters they were intermediate between *Prunus spinosa* and *P. cerasifera*. Rybin succeeded, moreover, in obtaining fertile hybrids between *P. spinosa* and *P. cerasifera*. One of them had a double chromosome number, namely 48, i. e. a number equal to that of the cultivated plum *(P. domestica)*, the chromosome number in *P. spinosa* being 16, in *P. cerasifera*—8, and in natural hybrids—24.

Comparative chemical investigation of *P. spinosa*, *P. cerasifera*, and of the cultivated plum. Comparative chemical investigation of the fruits of *P. spinosa*, *P. cerasifera* and of the cultivated plum furnished us valuable data which confirm the above hypothesis on the origin of *Prunus domestica*. Our investigation was conducted during two years (1936 and 1937) on the rich material of the Maikop Experiment Station of the All-Union Institute of Plant Industry. About 50 varieties of cultivated plums (*P. domestica*), 12 varieties of *P. insititia*, 27 forms of *P. cerasifera*, and 8 forms of *P. spinosa* were subjected to analysis.

Our study has proved that each of the species named above has a chemical composition of its own. Thus, *P. spinosa* is characterized by a low amount of saccharose, although comparatively rich in total sugar content. In *P. spinosa* citric acid is entirely missing, and tannic substances are abundant even in mature fruits. *P. cerasifera* is conspicuously different from *P. spinosa*. Variation in total sugar content is considerable and the amount of saccharose is high (almost in all the specimens studied it composed not less than 50 per cent of the total sugar content). It is also distinguished by high acidity; the fruits contain up to 7.7 per cent of citric acid (calculated for the dry weight). Tannic substances are absent even in unripe fruits of *P. cerasifera*. All the varieties of the investigated cultivated species (*P. domestica* and

P. insititia) combine the chemical properties of the two wild plums. In the total amount of sugar the cultivated plums leave far behind *P. spinosa* and *P. cerasifera*. The percentage of saccharose ranges widely: there are varieties of the type of *P. spinosa*, which are poor in saccharose, and such rich in saccharose, even approaching to the richest forms of *P. cerasifera*. The varieties of cultivated plums combine, finally, citric acid and tannic substances, the characteristic chemical components of both *P. spinosa* (tannic substances) and *P. cerasifera* (citric acid). The amount of these substances varies very much in the cultivated plums. Below are given the extremes for the four species mentioned.

Table 1

Amount of Sugar, Citric Acid and Tannic Substances in Various Species of Plums

Species	Year	Total sugar amount	Saccharose % to the total sugar	Total acidity*	Citric acid determined after pentabromaceton method	Tannic substances
		In % to the moist weight			In % to the dry weight	
P. spinosa	1936	5.9 —6.9	8—25	1.6—1.8	0	5 —10
» »	1937	5.04—5.95	3.5—13	1.6—1.7	0	4.5—12
P. cerasifera	1936	2.25—5.42	50—87	2.1—3.9	1.5—5.0	0
» »	1937	1.6 —7.7	35—85	1.4—4.1	1.5—7.7	0
P. insititia	1936	4.6—10.6	34—62	0.4—2.5	0—3.5	1.5—4.0
» »	1937	8.5—10.2	10—58	0.4—1.3	0—2.0	1.8—5.0
Cultivated plum	1936	5.5 —13.1	26—73	0.5—2.1	0—7.3	—
» »	1937	4.44—13.04	22—72	0.4—2.1	0—7.2	Traces

As may be seen from the table the limits of variation of the chemical composition are rather close to each other in the two years; this is considered as a chemical proof of the origin of the cultivated plum. A larger amplitude in *P. cerasifera* in the year 1937 should be explained by the fact that a considerably larger number of forms was studied than in 1936.

On the variation of the chemical composition of plums during ripening. Comparative chemical study of the fruits at different stages of ripening similarly confirms the fact that cultivated plums combine the chemical characteristics of *P. spinosa* and *P. cerasifera*. The total amount of organic acids in the fruits of *P. spinosa* varies within narrow limits during ripening; on the contrary, it increases twice during ripening in *P. cerasifera*. In cultivated plums the total acidity clearly drops towards the end of ripening, sometimes even more than twice. Citric acid is absent in *P. spinosa* during the whole development of the fruit. *In P. cerasifera* citric acid constitutes a considerable fraction in the green

* As calculated for citric acid.

fruits (up to 14% to the dry weight), and gradually drops towards the end of ripening. In cultivated plum there are varieties possessing in green fruits a small amount of citric acid, as well as such characterized by a rather high percentage; the variation curve drops here more conspicuously than in *P. cerasifera*. Tannic substances are absent in ripe fruits of *P. cerasifera* as well as in unripe ones; they are very abundant (1.31% of the moist weight) in the fruits of *P. spinosa*, and their amount drops with ripening. There are varieties of the cultivated plum which are rich in tannic substances in immature fruits (up to 13% of the dry weight). This is especially typical of the varieties of *P. insititia*. At the same time there are varieties of *P. domestica* very poor in or even entirely lacking tannic substances even in unripe fruits.

It should be mentioned that regularities of the storage of chemical substances in the fruits of plums in certain cases can be sufficiently cleared up only upon study of the chemical composition of fruits at different stages of ripening. In the ripe fruits of cultivated plums certain characteristics (citric acid, tannic substances) may be absent, while they can be observed in the green fruits. With regard to their chemical composition, the fruits of cultivated plums at different stages of ripening are very similar now to *P. spinosa*, now to *P. cerasifera*: these observations on the chemical changes during ripening are also in agreement with the theory of the hybrid origin of the cultivated plum.

Below is a Table 2 which illustrates the variation of the chemical composition of the fruits of plums at various stages of ripening.

Table 2

Variation of the Chemical Composition of the Fruits of Plums During the Process of Ripening Per Cent of the Moist Weight

Substances	Wild *P. spinosa*, No 2					*Prunus cerasifera*, No 5					Cultivated plum Reine-Claude Altane				
	20. VI	1. VII	14. VII	12. VIII	28. VIII	16. VI	27. VI	11. VII	21. VII	10. VIII	7. VI	20. VI	5. VII	18. VII	17. VIII
Total sugar . .	1.9	1.68	2.13	3.95	5.04	0.54	0.90	1.15	1.32	3.4	1.97	2.2	2.3	6.6	7.7
Saccharose . .	0.45	0	0.26	0.45	0.18	0	0.27	0.48	0.49	2.87	0.45	0	0.36	1.74	3.3
Total acidity .	1.8	1.66	1.22	1.25	1.65	2.0	2.84	2.77	3.0	3.4	2.3	1.38	0.50	0.46	0.69
Citric acid . .	0	0	0	0	0	0.57	0.83	0.98	0.73	0.42	0.9	0.81	0.2	0.42	Traces
Tannic substances	0.6	1.05	1.31	0.8	0.7	0	0	0	0	0	Traces	Traces	0	0	0

In conclusion we give some data on the analysis of the hybrids between *P. cerasifera* with other species of plums. The analysis of the chemical compositions also supports the hypothesis of the hybrid origin of these forms, which permits us to draw the conclusion on the hybrid origin of the cultivated plum from chemical data. The fruits of the variety «Marianna» (a hybrid between *P. cerasifera* and the american plum) in spite of ther high acidity and the presence of citric acid in them— characters typical of *P. cerasifera*, do not belong to the type of the latter owing to an insignificant accumulation of saccharose during ripening.

Table 3

Date of analysis	15.VI	1.VII	15.VII	28.VII
Per cent of saccharose	0.21	0.05	0.05	0.29

Another hybrid which was analysed by us (No 17, Shuntuk), is considered by taxonomists morphologically to be a hybrid between *P. cerasifera* and the cultivated plum. The chemical composition of the fruits also supports the hybrid origin of this tree. In the process of sugar accumulation and particularly that of saccharose, this hybrid is close to *P. cerasifera*, but its fruits contain tannic substances which are lacking in *P. cerasifera*; accumulation of citric acid occurs in the same manner as in cultivated plum.

Table 4

Date of analysis	13.VI	3.VIII
Citric acid, %	0.4	0.25
Tannic substances, %	0.3	0

Further detailed study of the biochemical differences among the species of plums will undoubtedly enable us to approach more closely the problem of the origin of plums. Crucial evidence should be expected from the chemical studies on the hybrids between *P. spinosa* and *P. cerasifera* which have been already experimentally produced at the All-Union Institute of Plant Industry.

Comparative chemical study of the forms of *P. spinosa*, *P. cerasifera* and of the cultivated plum conducted on mature fruits as well as at different stages of ripening, enables us to consider the suggestion of geneticists that cultivated plums have originated in the course of hybridization of *P. spinosa* and *P. cerasifera* to be supported also by the evidence of chemical nature.

Biochemical Laboratory.
Institute of Plant Industry.
Leningrad.

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GENETICS

TETRAPLOIDY IN *LINUM* INDUCED BY HIGH-TEMPERATURE TREATMENT OF THE ZYGOTE

By A. N. LUTKOV

(Communicated by N. I. Vavilov; Member of the Academy, 22. II. 1938)

Since the publication of the results of the successful experiments on inducing polyploidy in maize by subjecting the ears to high-temperature treatment after different lengths of time subsequent to pollination (1), tetraploid forms have been obtained by temperature treatment of the fertilized egg-cell only in a few other plants, viz., wheat, rye, clover, and barley (2—4). The chief cause of the comparatively low effectiveness of this method in experiments conducted prior to recent months undoubtedly was the lack of precise embryological data, i. e., of precise knowledge as to the exact time of the beginning of the process of fertilization and as to the development of the embryo during the first stages.

In 1937 the writer commenced experiments on doubling the chromosome number in several forms of fiber and seed flax by high-temperature treatment of the zygote. Simultaneously with these experiments embryological investigations were carried on.

In the present paper we give the preliminary results of the experiments made with one of the strains of Mediterranean large-seeded flax, K. 302, from Marocco, *Linum usitatissimum* L. ssp. *mediterraneum* Vav. and Ell. ($2n=32$). Sowings were made in the greenhouse in April, 1937.

A study of pollen germination from the moment the pollen was placed on the stigma, when plants of the given strain were kept after pollination under conditions of a medium temperature of 23—25°C, showed rapid growth of the pollen-tubes through the style. One hour after pollination a number of the tubes had penetrated 40—48 per cent of the length of the style, two hours after 88—90 per cent of its length, and two and one half to three hours after—many had already reached the end of the style.

However, despite such a rapid growth of the pollen-tubes through the style, development of the embryo-sac does not begin for some time thereafter. As our preliminary embryological investigations have shown—in conducting which we were aided by the valued suggestions of V. A. Poddubnaya-Arnoldi and which we made by a short-cut method, i. e., by studying sections of the ovule in a drop of aceto-carmine—12, 18, and even 20 hours after pollination the process of fertilization has not yet begun. The first divisions of the fertilized egg-cell and the formation of a two- or three-celled embryo, under a temperature of 25°C, were found to take place, with more or less constancy, 30—33 hours after pollination. Four- and six-celled embryos under these

conditions were formed several hours later (Fig. 1). A change in temperature conditions had a marked effect on the rapidity of development of the embryo. Thus, at a temperature of about 22°C two- and four-celled embryos developed only 36—38 or even 40 hours after pollination, and six- and eight-celled embryos only 42—46 hours after. A change in the temperature under which the plants are grown prior to pollination also markedly affects the rapidity of development of the embryo.

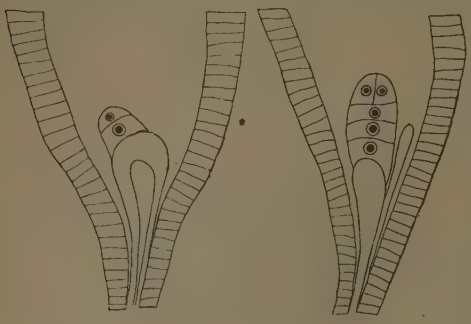

Fig. 1. Two- and five-celled flax embryos.

On the basis of the embryological data obtained a number of experiments were conducted on subjecting plants of the given strain of seed flax to high temperature, ranging in different cases from 40 to 48°C. The plants, grown after pollination in a special chamber with a temperature ranging between 22 and 25°C, were—after different intervals of time following pollination and after examination of sections from ovules of different flowers—quickly transferred to a high-temperature chamber for a period of 30 minutes to 1½

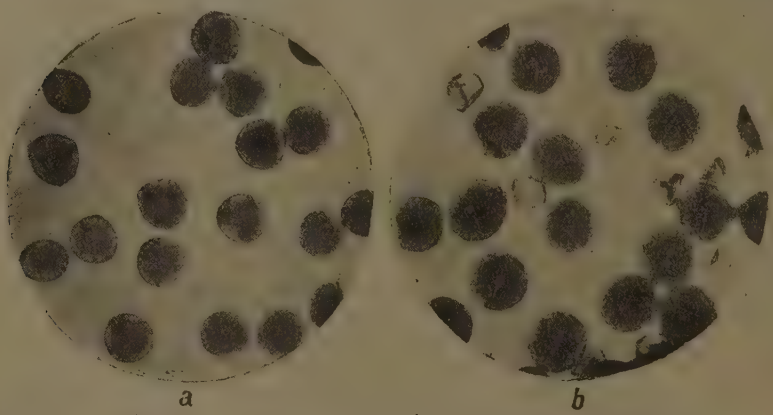

Fig. 2. Pollen grains: *a*—of diploid flax; *b*—of tetraploid flax.

hours. Afterwards the plants were grown under ordinary conditions until maturity.

In August 240 seeds, collected from treated plants, were sown.

During the flowering period, from September 22 to October 27, 190 plants were studied as regards size of pollen grains and also as regards fertility or partial sterility. This investigation showed that, while almost all the plants had entirely normal pollen, the diameter of the grains ranging from 61.2 μ to 76.5 μ, one plant (7.83—4), which had begun flowering on October 13, possessed pollen grains varying greatly in size. A considerable portion (47.5%) of the pollen grains of this plant proved to be abortive. The diameter of the abortive grains ranged from 22.95 μ to 61.2 μ, while that of the plump grains ranged from 61.2 μ to 114.75 μ. Most of the grains had a diameter of 91.8 μ (Fig. 2). These large pollen grains were readily distinguishable by their size

from ordinary ones. A cytological analysis showed that this plant (7.83—4) had in its somatic cells 64 chromosomes, i. e., it was a tetraploid (Fig. 3).

Morphologically this tetraploid plant, which arose as a result of one hour's treatment with high temperature (46°C) begun 33 hours after pollination, was also markedly distinct from the other plants of strain K. 302. It also differed from the other four plants in Family 7.83, to which it belonged. Its flowers, for instance, were considerably larger (Fig. 3). Thus, flowers of diploid plants, according to rough measurements made in October, did not exceed 22—23 mm in diameter, while those of the tetra-

Fig. 3. Flowers and somatic plates of chromosomes of diploid and tetraploid flax.

ploid attained 28 mm. Moreover, some flowers of the tetraploid had six, instead of five, petals. Separate parts of the flower were also altered to a considerable degree.

As regards the length of the stem, measurements of all plants of Family 7.83 showed that, while in the tetraploid form the length of the stem just prior to flowering was 77 cm, in three of the diploid plants of this family it was less, but in the fourth diploid it attained 85 cm. The length of the stem of the other diploid plants also varied greatly, ranging from 30 to 100 cm. However, these data may be regarded as only approximate, since the late sowing (in August) caused a considerable increase in the height of the plants, due presumably to the higher temperature at the time of the beginning of the vegetative period and to the lack of sufficient sunlight subsequently.

As regards thickness of stem, the tetraploid plant exceeded the diploids, its stem having a diameter of 2.9 mm at a height of 50 cm from the base.

However, at the point where branching began and below this point its diameter was slightly less than that of a few of the diploid plants. However, these data can serve only as an index. It is necessary to make further analysis of the tetraploid form by its reproduction and comparison with the initial diploid form under normal conditions of vegetation. Such an analysis is fully possible, since at the end of the vegetative period the tetraploid plant bore six bolls.

Of very great interest likewise would be a detailed anatomical study of the tetraploid form obtained, but even now it may be stated a priori that, as a result of doubling the chromosome number, all the anatomical elements, particularly the phloem cells, must be considerably enlarged in tetraploid as compared with diploid flax. A direct indication of this is the marked change in the anatomical elements in experimentally produced tetraploids of other plants, and also the anatomy of a haploid flax plant obtained by us as a result of temperature treatment*. All the anatomical elements of the haploid plant are extremely diminished in size as a result of the diminution in chromosome number.

In addition to the tetraploid plant with a doubled chromosome number in all its tissues, we found three chimaerical plants. In one of them chromosome doubling occurred only in occasional root-tips, but in the other two entire patches of stem tissue proved to be tetraploid. The pollen grains of some of the flowers of these two chimaeras greatly varied in size, just as did those of Plant 7.83—4, a high percentage of them likewise being abortive.

Further analysis of the tetraploid form of this Mediterranean flax—the diploid form of which is characterized by both high oil-content and high-quality fiber, but has not been grown commercially because of the insufficient length of that portion of the stem suitable for textile fiber—will elucidate its possible significance for practical breeding purposes.

Laboratory of Genetics.
Institute of Plant Industry.
Pushkin (near Leningrad).

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ENDOCRINOLOGY

ANTIGONADOTROPIC SERUM

By J. M. KABAK

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th as been shown by us earlier (5) that the unsusceptibility to gonadotropic preparations of human origin acquired by animals after repeated injections of such preparations possesses species specifity, i. e. that those antagonistic substances, which, as has been demonstrated by Collip (1), appear in the circulation of such «immune» animals, are species specifically directed.

A similar conclusion has been drawn from their researches by Gustus, Meyer and Dingle (4), by Flühmann (2), by Zondek and Sulman (8). However, Gegerson, Clark and Kurzrok (3) and Parker and Rowlands (6,7)—the latter in a series of studies—have presented data of an opposite character.

To analyse these contradictions we studied sera, obtained from different animal species by means of their immunization with various gonadotropic preparations.

As test-object we used infantile female rats, to which 1 cm³ of the serum under test was administered daily subcutaneously on three consecutive days simultaneously with the gonadotropic preparation, which was injected also subcutaneously on the opposite side of the body. The rats were autopsied 5 days after the first injection and their response evaluated by the weight of the ovaries and uterus. All the quantitative data cited in the present article are expressed in «rat» units.

The Serum of Rats Treated with Prolan* for a Long Period of Time (Serum 1-S-1)

The rats, whose serum was utilized in these experiments, were injected 15 units of prolan daily during 3 months or longer.

3 cm³ of S-1 suppress the effect of up to 150 units of prolan and that of 9 units of a gonadotropic preparation from human hypophysis. No antagonistic properties whatever are shown in respect of gonadotropic preparations from sheep and pig hypophysis as well as in that of preparation from the serum of a pregnant horse. A certain interaction with the latter is, however, manifested. Thus, after a simultaneous injection of 1 unit of the preparation from the pregnant horse serum and of 3 cm³ of S-1 no gonadotropic response was manifested in the weight of the ovaries, while the weight of the uterus increased quite markedly for this dose.

* Gonadotropic preparation obtained from the pregnant women urine.

The Serum of Rabbits Treated with Prolan for a Long Period of Time (Serum 2-S-2)

The rabbits, whose serum was used for these experiments, were injected 150 units of prolan daily for 3 months and longer. The blood was obtained from the ear vein, 15—20 cm^3 being taken from every animal at a time.

S-2 proved very active in regard to gonadotropic preparations of human origin. 3 cm^3 of S-2 counteracted the effect of these preparations in all the experiments, in which up to 750 units of prolan or 90 units of the preparation from human hypophyses (higher doses have not as yet been tested) were injected. At the same time S-2 does not manifest any antagonistic properties whatever against gonadotropic preparations from sheep and pig hypophyses. 3 cm^3 of S-2 injected together with the gonadotropic preparation from sheep and pig hypophyses show even an effect of augmenting their action. Thus, e. g. the injection of 12 mg per rat of pig hypophyses dried by acetone increased the weight of the ovaries to 14 mg and that of the uterus to 49 mg on the average. After a simultaneous injection of the same preparation and of 3 cm^3 of S-2 these weights were 20 and 95 mg respectively.

4.5 mg of the preparation from sheep hypophyses provoke a weight increase of the ovaries to 23 mg and of the uterus to 70 mg. The same dose of the preparation introduced simultaneously with 3 cm^3 of S-2 increased these weights to 60 mg and 66 mg on the average respectively.

In respect of the gonadotropic preparation from the serum of a pregnant horse S-2 shows only traces of an antagonistic action: 3 cm^3 of S-2 can suppress the action only of one unit of this preparation.

The Serum of Rabbits Treated with a Gonadotropic Preparation from Human Hypophyses for a Long Period of Time (Serum 3-S-3)

Rabbits were injected daily with an alkaline extract of human hypophyses dried by acetone, filtered through a Seitz filter, in quantities corresponding to $^1/_{20}$ of hypophyse per rabbit and day. The serum of these rabbits (S-3), tested together with various gonadotropic preparations, showed the following results. S-3 is antagonistic to preparations of human origin: 3 cm^3 of S-3 can counteract the effect of more than 60 and less than 150 units of prolan or of about 7 units of the gonadotropic preparation from human hypophyses. At the same time S-3 shows no antagonistic action whatever towards gonadotropic preparations from pregnant horse serum and from pig and sheep hypophyses.

The simultaneous injection of S-3 with the gonadotropic preparation from pig hypophyses shows an enhancing effect. Thus, e. g. after a simultaneous injection of 3 cm^3 of S-3 together with 12 mg of the preparation from pig hypophyses the weight of the rats' ovaries averaged 65 mg and that of the uterus 80 mg.

After the injection of the same amount of this preparation without serum the average weight of ovaries and uterus was 14 and 49 mg respectively.

The Serum of Rats Treated with the Gonadotropic Extract from Human Hypophyses for a Long Period of Time (Serum 4-S-4)

Rats were daily injected with an alkaline extract of acetone-dried human hypophysary tissue in amounts corresponding to $^1/_{40}$ of a hypophysis per rat and day during 2.5—3 months. Experiments with S-4 are in perfect agreement with the results of the series of experiments described above,

though they were less detailed than these. S-4 shows an antagonistic action both in respect of prolan (3 cm³ of S-4 can counteract up to 50 units of prolan) and of the gonadotropic preparation from human hypophyses (3 cm³ of S-4 can counteract 20 units). At the same time 3 cm³ of S-4 shows only traces of an antagonistic action towards gonadotropic preparation from pregnant horse serum and none whatever towards the one from sheep hypophyses.

The Serum of Rabbits Treated with Bovine Hypophysary Extract for a Long Period of Time (Serum 5-S-5)

To rabbits were daily administered injections of Seitz-filtered alkaline extract of acetone-dried bovine hypophyses, during 3 months or longer. The amount of extract introduced with each injection corresponded to 0.5 g of fresh tissue. S-5 shows an antagonistic action towards gonadotropic preparations of human origin (3 cm³ of S-5 counteracts >10 and <50 units of prolan and up to 3 units of the hypophysary preparation). At the same time S-5 shows no antagonistic action whatever towards gonadotropic preparations from pregnant horse serum and from pig and sheep hypophyses.

When 3 cm³ of S-5 is injected simultaneously with a minimal dose of the preparation from pig hypophyses an enhanced effect of the latter is clearly manifested.

Conclusions

The results of all the experiments described above are summed up in the following table, in which all the quantitative data have been expressed in terms of the action of 1 cm³ of serum.

Donor	Preparation used for the pre-treatment of the donor	Antagonistic activity of 1 cm³ of serum to preparations of various origin (rat units)				
		Pregnant woman urine	Human hypophyses	Sheep hypophyses	Pig hypophyses	Pregnant horse serum
Rat	Prolan	50	3	No counteraction	No counteraction	No counteraction
»	Human hypophysary extract .	17	6	Ditto	Ditto	Traces (1/3)
Rabbit	Prolan	>250	>30	Increase	Increase	Traces (1/3)
»	Human hypophysary extract .	> 20<50	2	No counteraction	Ditto	No counteraction
»	Bovine hypophysary extract .	> 3<17	1	Ditto	Ditto	Ditto

The following conclusions can be made:

1. Our earlier conclusion, that the antigonadotropic properties arising in the sera of animals treated chronically with gonadotropic preparations from human hypophyses or urine possess strict species specifity. These sera do not manifest any antagonistic action in regard to gonadotropic preparations of a different origin (from other animal species).

2. Sera obtained by means of chronic injections of bovine hypophysary extracts, do not possess so strict a species specifity. They are active also against gonadotropic preparations of human origin. However, in this case

as well one cannot deny the presence of a species specifity altogether, since these sera have no antagonistic properties in regard to gonadotropic preparations from pig and sheep hypophyses and from pregnant horse serum.

3. The results of experiments with the sera of animals, treated with human hypophysary extract, do not corroborate the assumption of Zondek and Sulman concerning «organ» specifity.

4. Attention is drawn to the fact, that some sera exert an augmenting effect on pig and sheep hypophysary extracts. These facts demand further study.

5. The results obtained in the present study contradict with the «antihormone» theory of Collip. The mechanism of the habituation to gonadotropic preparations may probably be explained by analogy with the well known immunologic reactions.

Saratov State University.

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AGROTECHNICS

ON A NEW METHOD OF TAU-SAGHYZ EXPLOITATION

By F. P. MAZANKO

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Latex in the latex vessels of rubber-bearing plants, root rubber plants including, is subject to the high pressure of parenchymatous cells surrounding the latex vessels. With the mechanical injuries and opening of the latex vessels latex is driven outwards through the places of opening and transported towards the places of injury. By a series of experiments we have established that with cutting the roots of tau-saghyz immediately below the crown the zone of the latex transfer through the latex vessels covers the area above a meter downwards from the place of the cut. Thus, if the roots of tau-saghyz, kok-saghyz, or krym-saghyz are cut immediately below the crown, latex flows abundantly to the outside of the cut surface and coagulates after a while, forming a swelling of rubber on the place of the cut. The latex vessels are corked up in this place by coagulated latex and the outflow of the latter ceases.

The loss of latex forced out is made up in the latex vessels at the expense of reserve substances contained in the roots [Mazanko ([1],[2])].

Thus, the tapped plants restore equilibrium on the 4th or 5th day. Their latex vessels are filled up with latex again and become subject to high turgor pressure of the adjacent parenchymatous cells. If by this time the swelling of rubber is removed from the surface of the cut or if cutting is renewed, the outflow of latex and rubber will be observed again in the place of the cut. Such tapping of the tau-saghyz roots preceded by removal of swellings increases the economical and biological yield of rubber per plant from 2 to 2.5 times compared with rubber reserve in the «untapped» plants. This is illustrated by the figures of Table 1.

Tapping was started at the time when the plants proceeded to the state of summer rest, their leaves being dried up; thus, no harm could have been done to them from the point of view of complete utilization of their active vegetation period.

After the maximum possible yield of rubber from tau-saghyz plantation, secured by means of tapping, had been established, we set ourselves the task of elucidating the following questions:

1) What is the optimum interval between tappings for obtaining the maximum yields of rubber from rubber plants.

2) What should be the system of tappings to secure the most protracted period of vital activity of plants.

3) Whether there exists a possibility to keep the plants in a state of vital activity after performing the cycle of tappings and to repeat these cycles on the same plants during several years.

Table 1

Changes in Absolute Rubber and Resins Contents of Tau-Saghyz in Connection with Tappings*

Material	Length of the root, cm	Average weight of absolutely dry root, g	Absolute quantity per root		
			Ruber, g	Resins, g	Total, g
Tau-saghyz planted in autumn, 1938. Control plants (untapped)	30	7.565	1.134	0.216	1.350
Tappings:					
in 2 days	30	6.876	2.163	0.378	2.541
» 4 days	30	7.893	2.328	0.407	2.735
» 6 days	30	9.466	2.669	0,320	2.989
» 8 days	30	7.491	1.959	0.365	2.324

We succeeded to answer these questions owing to research work done during 1936 and 1937.

The technique of the experiments was as follows.

Prior to tapping operations the roots of the tau-saghyz plant were set free from the soil to the depth of 3—4 cm. On the upper portions of the roots of one group of the experimental plants there were made lateral cross-cuts as deep as one third of the root diameter. In the other group of the experimental plants roots were entirely cross-cut below the crown, the overground parts being completely removed. The cutting was done by means of a sharp razor. The latex effused coagulated on the surface of the cut, forming a swelling. At certain intervals between tappings (1 day, 2 days, 3 days, 4 days, etc.) the swellings were removed. After their removal a second abundant effusion of latex was observed on the surface of the cut. In case the second effusion of latex was insufficient, a new cut was made, 1—2 mm from the first one. After the tappings had been performed some of the experimental and of the control plants were dug up for determination of the amount of latex remaining in the roots. The rest of the experimental plants after hibernation regenerated the overground parts and had their vegetation period during the whole of the next summer. When the plants proceeded to the state of rest (by the end of September) they were tapped again. The tappings were effected according to the scheme: once a day, once in 2 days, in 3 days, in 4 days, etc. From all these variants the most practically acceptable one proved to be that of 4-day intervals. The variants with more frequent tappings caused the plants to take rot as early as on the 15th—20th day and to die away.

It appears most probable that in the given case the same phenomenon has been observed as recorded by Frey-Wyssling (4) for Hevea. From his point of view the diseases of Hevea arise owing to the intensive tapping of latex in connection with which the turgor pressure in latex vessels drops, while their absorbing power increases considerably. Thus, a most rapid absorption of water from the parenchymatous cells adjacent to the latex vessels is made possible which results in a rotting and dying out of these cells.

* Tapping implies either removal of a swelling or renewal of a cut resulting in latex outflow.

Hence the schemes involving tappings performed more frequently than at 4-day intervals proved to be unfit, since they brought death of many of plants. As to the total productivity of plants, the records show a maximum for the scheme of 4-day intervals; the tappings may be effected 10 to 12 times during 40—50 days.

The advantages offered by the scheme of tapping at 4-day intervals are illustrated by Table 2.

Table 2

Material	Number of tappings	Date of the 1st tapping	Amount of rubber obtained per 100 plants, g Tappings in		
			2 days	4 days	6 days
Tau-saghyz planted in autumn 1933	11	9.VI	84.6	124.6	92.6

To estimate the pertinence of the method tested under industrial conditions an experiment involving 12 000 plants was conducted on the tau-saghyz plantations. Comparative evaluation of the productivity of the plantations (for 1936 and 1937) exploited after 2 methods, the usual digging up of roots, and the tapping method, showed that the application of the latter, without destroying the plantation by digging, made it possible to obtain yields of rubber 2—2.5 times exceeding that obtained from the digged up roots. It should be added, moreover, that the swellings present a highly valuable industrial product, consisting of pure rubber with but a slight admixture of resins. The productivity of a valuable plantation (computed for 150 000 plants per hectare) which was exploited after different methods, is illustrated by Table 3.

Table 3

Material	Number of plants per hectare in thousands	Amount of rubber per hectare, kg			Amount of resins per hectare, kg			Total amount of rubber and resins		
		In swellings	In root-vessels	Total	In swellings	In root-vessels	Total	In swellings	In root-vessels	Total
1. Tau-saghyz planted in 1933. Controls taken on Nov. 14, 1936	150	—	103.3	103.3	—	19.9	19.9	—	123.2	123.2
2. Tau-saghyz plants tapped in 1936. Oct. 7, Nov. 14 . . .	150	122.4	111.1	233.5	7.5	20.7	28.2	129.9	131.8	261.7
3. The same tau-saghyz plants repeatedly tapped in 1937 . .	150	175.57	168.6	344.27	3.58	31.35	34.93	179.25	199.95	379.2
Controls taken on Dec. 4, 1937 . .	150	—	145.8	145.8	—	27.75	27.75	—	173.55	173.55

After the second series of tappings in 1937 the plants entirely maintained their viability and proved fit for the third cycle of tappings in 1938.

It may be seen from Table 3 that a four years old tau-saghyz plantation with 150 000 plants per hectare has after 3 cycles of tappings, made during 1936 and 1937, 309 kg of rubber in swellings, without taking into account the amount of rubber remained in the roots. The same plantation of tau-saghyz if exploited after the method of digging up the roots to the depth of 20 cm would yield only 173.55 kg of rubber per hectare. The rubber thus obtained would be of worse quality, since the extraction at works impairs its initial high quality. Estimations show that even with manual tapping of plants and removing of swellings the expenditures may be expressed by a rough figure of 15—16 thousand roubles per ton of rubber, which seems satisfactory enough to introduce this method into wide economical practice.

It will prove to be still more expedient in case maximum mechanization of tapping proceedings will be secured.

The results obtained make it obvious that the above method should be widely adopted by our industrial practice, which was confirmed too by the experiments conducted on a large scale in the farms of the Trust «Kauchukonos» in 1937.

Conclusions

1. The intense rubber formation in the latex vessels of tau-saghyz and its intense regeneration after effusion, makes it possible to adopt a rational system of tau-saghyz exploitation by means of tapping the roots cut below the crown.

2. Of all the methods of tapping tested that of cutting roots below the crown at the period of transition to the state of summer rest and leaf drying proved the best.

3. This method makes it possible to perform about 8—10 removals of rubber swellings during the season, and secures the maximum yield of high quality rubber as compared with other schemes involving the same number of tappings; at the same time it keeps the plants viability unaffected and admits subsequent tappings during the next years.

4. After the method of tappings we may during several years obtain about the same amount of rubber as is contained in the roots of untapped plants.

5. All above said points to the necessity of a wide application of the method of tau-saghyz tapping both in large scale experiments and in wide industrial practice. Our next task as to improve the method advanced by the author will be to devise a most complete mechanization of the tapping proceeding.

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PLANT PHYSIOLOGY

ON RUBBER FORMATION IN TAU-SAGHYZ

By F. P. MAZANKO

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The question of the locus of rubber formation in rubber plants may be viewed from two different points. Thus, rubber formation may be considered closely related with the assimilative activity of the plant, rubber being one of the first products of photosynthesis, forming only in the assimilating tissues of the leaves. After being formed in the leaves rubber flows rapidly down to the root parts where it is stored up in the latex vessels [Novikov (1)]. If this is the case, latex vessels are not the loci of rubber formation, but merely the ducts of its transportation and the places of storage.

The other interpretation assumes that rubber may be formed in the very latex vessels provided the necessary reserve of organic matters be present [Bobylioff (2), Popovici (3)]. Inulin, for instance, which by the period of autumn rest is abundantly (up to 50%) accumulated by the roots of such plants as tau- and kok-saghyz, is believed to be one of the substances preceding rubber formation. Part of this reserve of inulin can be converted into rubber (if one admits that rubber formation is not necessarily connected with photosynthesis). The solution of this problem is not only of theoretical but of great practical importance too. If we have studied the conditions of rubber formation in the latex vessels, we can look forward to a possibility of directing this process towards a maximum transformation into rubber of the enormous inulin reserve contained in the roots of the rubber plants, not only during the periods of intense assimilating activity of the leaves, but also during the periods of decrease and cessation (period of rest).

Our purpose was to elucidate the question of rubber formation in the latex vessels of the roots of tau-saghyz, without the reference to the assimilating activity of the leaves.

The work was performed at the Atabaevskaya Experimental station belonging to the system of the All-Union Research Institute of Rubber and Gutta-Percha. In our experiments we took into consideration that the restricted capacity of latex vessels already filled up with rubber could prevent the process of additional rubber formation no vacant place being present [Nichiporovich (4)]. Thus, the additional accumulation of rubber in the root latex vessels without the participation of the leaves could have been limited not because of the vessels being unable to produce it, but owing to the reason referred above.

In order to avoid this following method was adopted in our experiments: trenches 50 cm deep were dug along the rows of a tau-saghyz plantation at

the third year of vegetation. By means of horizontal diggings the roots on the depth mentioned above were cut crosswise. After this operation the trenches were carefully covered and the plants were watered. When the plants had entered the state of summer rest, they were repeatedly tapped from the lower part of the roots through the periodically renewed cuts which resulted in an effusion of latex and a partial discharge of the latex vessels.

As the plants were in the state of summer rest, bearing dry leaves, we could presume that the regeneration of the rubber after tapping could proceed only at the expense of its additional formation in the latex vessels. The control plants were subjected to the same treatment as those under test, except periodical tappings, and might produce as much rubber as could be contained in the latex vessels.

Summing up the amount of rubber obtained from effusions and the rubber remained in the roots by the end of the experiment, we should get higher indices for the experimental plants if additional quantities of latex would have been formed after tapping.

The work with periodical tappings was continued up to September, after which the roots were dug up, and at the same time the control plants also were gathered.

The results of the two parallel experiments are brought together in Table 1.

Table 1

Experiment	Material	Depth the plants were cross-cut and dug up, cm	Average weight of an absol. dry root, g	Rubber amount per root, mg	Resin amount per root, mg	Increment			
						Rubber		Resins	
						mg	%	mg	%
1	Tau-saghyz controls, Oct. 8 . .	50	2.241	216	61	—	—	—	—
	Experimental plants, Oct. 8 .	50	2.024	257	77	41	19	16	26.2
3	Controls	50	1.628	160	37	—	—	—	—
	Experimental plants	—	1.534	208	43	48	30	6	16.2

As may by seen from the table, during the period of summer rest the plants of tau-saghyz were able to form additional quantities of rubber in case their latex vessels were periodically discharged by tapping, thus affording the possibility of rubber regeneration.

It should be noted that the yield of rubber tapped from the lower parts of the roots at the depth of 50 cm was practically very poor, as in this zone the roots were of small diameter. It seemed probable that the rate of rubber regeneration would considerably depend on the discharge intensity of the latex vessels.

Thus, a more intense effusion should involve a more intense regeneration process.

To secure these conditions, supplementary experiments were carried out in which the tapping was systematically done not in the lower but in the upper portions of the roots, the cuts being made immediately below the crown. In this case the overground parts of the plants were removed and therefore the participation of the leaves in rubber formation was eliminated. Besides this, tapping experiments were performed after the

same method as the preceding one. As it might have been expected, the last series of experiments gave more accurate results, which are summarized in Table 2.

Table 2

Experiment	Material	Depth the plants were cross-cut a. dug up, cm	Average weight of an absol. dry root, g	Rubber amount per root, mg	Resins amount per root, mg	Increment			
						Rubber		Resins	
						mg	%	mg	%
1	Controls at the beginning of the experiment, Sept. 4	40	3.606	450	105	—	-	-	—
	Controls by the end of the experiment, Sept. 4	40	3.715	532	107	82	18.2	2	1.9
	Experimental plants, Nov. 4	40	3.855	891	117	359	67.5	10	9.3
2	Controls by the end of the experiment, Nov. 4	40	3.749	456	98	—	—	—	—
	Experimental plants	40	3.853	801	124	345	73.5	26	26.6

The figures of this table show that the control plants were during this period in the state of rest, yet they gave some increment of the rubber content of the roots. This increment was, however, considerably less than that in the experimental plants which were systematically tapped.

Thus, our figures prove irrefutably the possibility of rubber formation directly in latex vessels without participation of leaves, provided the latex vessels be periodically discharged from the portion of their latex and rubber contents. The data obtained suggest the following conclusions which are of great importance for the practice of rubber plant exploitation in this country.

After the method adopted in the USSR for obtaining rubber by means of digging up the roots and their subsequent factory treatment for rubber extracting, only a small part of potential efficiency of the rubber plants is turned to use. Their biochemical mechanism affords the possibility of formation of far greater amounts of rubber than that contained in the latex vessels. This fact was supported by many other authors too. Thus, according to Lock's (5) estimations, a Hevea tree during three years of gathering rubber by means of tapping gives an amount of rubber 8 times as great as that contained in the bark prior to tapping operations. Vendixen (5) calculated that a Hevea tree containing a total of 6 kg rubber gives with daily tappings 20 kg of rubber per annum. Kiselev (6) established that by means of an artificial formation of rubber swellings in Chondrilla plants (through mechanical injuries) and their subsequent removal, it was possible to gather the amount of rubber which exceeded by 23—25% that which is stored up in the tissues of the plant under natural conditions.

From the above it appears expedient to devise some essentially different method of our rubber plant exploitation, which would afford the possibility of periodical gatherings of rubber by means of letting it out together

with latex from the latex vessels without disturbing the normal vital processes of the plants.

In agreement with this the system of exploitation of root rubber plants, inclusive of tau-saghyz, should be based on the rubber regeneration in the latex vessels at the expense of reserve matters contained in the roots.

Our experimental work conducted in this direction and checked in 1937 by wide industrial experience, has fully confirmed the economic efficiency of the new method of tau-saghyz exploitation by means of tapping.

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PLANT PHYSIOLOGY

RESPONSE OF UNRIPENED WHEAT GRAIN TO YAROVIZATION EFFECT OF CHILLING

By T. J. ZARUBAILO

(Communicated by N. I. Vavilov, Member of the Academy, 25. II. 1938)

As in experiments of Kostiučenko and Zarubailo ([1], [2]) the ripening of wheat seed at low temperatures gave results analogous to those obtained with artificial pre-sowing yarovization, these experimenters came to the conclusion that seeds which are and have not completed ripening on the mother plant are as able to pass the stage of yarovization as the seeds which have begun germinating.

Such conclusion required further experiments which would prove that the effect obtained in rearing seads at low temperatures was in fact a result of direct action of these temperatures on the ripening seed and not an after-effect of conditions under which the mother plant has grown.

Kostiučenko and Zarubailo experimented on grains of the same wheat variety reproduced in different localities (south and north) as well as on grains that had ripened in the same locality (Pushkin Experimental Station, Plant Breeding Institute) but at different dates, therefore, at different temperatures.

All plants whose seeds were taken for experiment had been subjected to similar temperature conditions until earing set in. After earing, during seed formation and ripening, these conditions differed greatly. This is clearly seen from the graph on Fig. 1, where the mean 10-day temperatures are given, referring to vegetation period of wheat plants whose seeds have been taken for experiment from remote localities, different in their geographical position and climate, such as Chibiny (67°44′ N. Lat.) and Kirovobad (40°41′ N. Lat.).

Comparing the curves we see that before and during the stage of earing there is no substantial difference between temperatures at Chibiny and Kirovobad, while these temperatures differ greatly during the stage of seed formation and ripening.

The latter circumstance undoubtedly accounts for the difference which was observed in the progeny of the plants of different origin, viz., the plant grown from seeds of Chibiny eared when sown in spring without or after a short (10-day) pre-sowing yarovization, whereas the same varieties from seeds of Kirovobad did not ear even after a 30-day yarovization.

A dualism in the state of a forming and ripening seed should be noted. Such seed is a part of the mother plant since it forms, ripens and feeds on the

latter, and yet it is already a new individual organism. As part of the mother plant the seed is not indifferent to actions which the plant suffers; as an individual organism it has its proper relation to environment. Thus, environment affects the seed in two ways: through the mother plant and immediately.

In which way has it produced its influence in the experiment mentioned? No doubt this has been an immediate influence since we may hardly believe the influence of environment conveyed through the mother plant to have left a sharper impression than that exerted immediately. The more so as during formation and ripening the seed is the most sensitive part of the plant, the

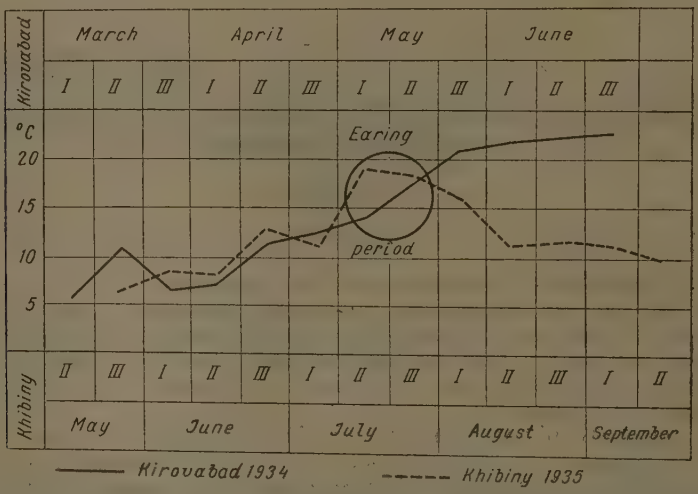

Fig. 1. Mean 10-day temperatures plotted for the vegetation period of wheat cultivated at Kirovabad and Khibiny during 1934 and 1935 respectively. The seeds of the crops of these years were used by Kostiučenko and Zarubailo for their experiments.

other parts growing coarse and becoming less and less sensitive. We have surely to suppose that the young and very sensitive organism, as a forming and ripening grain is, directly suffers the action of environmental factors, being moreover in immediate contact with them, and by no means through the mother plant which is by far less sensitive to external influence.

To prove this we have carried out the following experiment.

In July 1936 one wheat plant «Ukrainka» sown in fall and being in the middle period of ripening was taken. Two ears of the plant were cut off and dried in the laboratory at a temperature of 20° C. The remaining three ears in the same period of ripeness were put into ice-house and left there for 30 days at a temperature of 3—6° C. The moist air of the ice-house kept the ears from drying and taken out after 30 days they were in the same fresh condition they had had when put there. Then these ears were also dried in the laboratory at about 20° C. The outward appearance of grains separated from them differed in nothing from grains thrashed out of the ears which had been dried in the laboratory immediately after being cut off.

In the early spring, on April 16, 1937, the grains of all the five ears were sown into pots, without being yarovized before sowing, and allowed to stand out of doors under a wire-net. All the seeds germinated simultaneously and in good time. But the further development of the plants showed great differences. The seeds kept in ice-house gave plants which began earing on July

9 and carried it through with one accord; the plants from the seeds dried at once gave no ears at all up to the end of autumn (Fig. 2).

Analogous results have been obtained by Gregory and Purvis (3) who placed ripening rye ears in water and kept them in a refrigerator at 1° C for a period of six weeks.

Fig. 2. Winter wheat Ukraïnka var. *Erythrospermum* sown in spring 1937 at the Pushkin Experimental Station, Plant Breeding Institute, harvested at the period of milk-ripeness and unyarovized previous to sowing. Left (two vessels); plants from seeds, dried at the laboratory at a temperature about 20°, immediately after harvesting. Right (two vessels): plants from seeds kept in ears for 30 days in a refrigerator at temperature from 3—6°.

Thus, the chilling of unripe seeds for a long period produced the same effect as the growing of seeds at low temperatures. Hence we may infer that in the latter case too, the low temperature affects the forming and ripening seeds immediately and not through the mother plant. The experiment proved more than once that not only germinating seeds are able to pass the stage of yarovization but also unripe seeds, from the middle period of ripening.

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PLANT PHYSIOLOGY

HORMONES OF GROWTH IN FORMATION PROCESSES. I. PHOTOPERIODISM AND CREATION OF GROWTH HORMONES

By M. Ch. ČAJLACHJAN and L. P. ŽDANOVA

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Recent advances in the study of growth hormones have revealed their important rôle in form-building processes of a vegetable organism. The last decenium has been a critical period in their study. From the coleoptiles of oat and other cereals—the classical subject of investigation—one proceeded to various plants of different age and instead of coleoptile tops used synthetic preparations as source of growth hormones. The use of these preparations was made possible owing to works of Kögl and his associates ([5],[6],[7]) who succeeded in separating the hormones «auxins» and established their chemical structure.

The experiments of Laibach and Fischnich ([8],[9]), Zimmermann and Wilcoxon ([11]) revealed the important part played by growth hormones «auxins» in the formation of callus and roots. Individual parts of plant gave roots when provided with auxins which had been rubbed in as paste or injected in them. In experiments of Cooper ([1]), Hitchcock and Zimmermann ([4]) the scions of shrub or tree plants, immersed in heteroauxin solutions, formed roots in a short time.

The influence of growth hormones on the formation of leaves has been reported by Laibach and Mai ([10]), who introduced heteroauxin into *Solanum Lycopersicum* and *Coleus thyrsoideus* by besmearing closed axillary buds and induced thus simplification of the leaf blade, coalescence of leaves, locking of the growing point in the petiole tissues and formation of ascidia (modified leaves).

The formation of shoots is also affected by the growth hormones and depends on their quantity. In experiments with *Circaea intermedia* carried on by Dostal and Hošek ([2]) the tops ready to flower, lopped from the plants and smeared with heteroauxin, gave stem shoots with delayed flowering instead of usual flowering shoots.

Interesting facts have been reported by Gustafson ([3]) who induced fruiting by means of growth hormones. He removed stamens from flower buds, clipped styles with stigmae and besmeared the cut surface on the style with a mixture of lanolin and heteroauxin the latter being sometimes replaced by chemically related substances, as indole propionic, indole butiric or phenyl-acetic acid. Under the action of those substances the unfertilized ovary develops into a fruit containing no seeds. Such ripe parthenogenetic fruits have been obtained in tomatoes, petunia, pepper and in *Salpiglossus variabilis*. In other plants these substances induced the growth of ovary to different sizes but no ripe fruits formed.

All the above facts show how important the rôle is, played by the growth hormones «auxins» in formation processes. They affect the formation of callus and roots, the structure of leaves and shoots, the development of seedless fruits; in other words, both vegetative organs and the pericarp of fruit are involved in the sphere of their influence. The cited works have not examined the rôle of hormones of growth in plants passing from vegetative growth to sexual development, from root, leaf and stem formation to flower-buds and flowers. Such study is, however, unavoidable to completely elucidate the part of auxins in the life of plants.

With this aim in view we investigated the change of growth hormones in plants subjected to sharp transitions from vegetative growth to reproduction due to external influences, and proved the effect of the synthetic preparations of hormones of growth (heteroauxin) which we introduced into the plants artificially. The whole of research work had been divided into three parts:

1) Changes in growth hormones due to photoperiodism—their response to change in the length of day light.

2) Changes in growth hormones due to yarovization.

3) The effect of the heteroauxin upon the growth and development of plants whose seeds have been treated with heteroauxin before sowing.

The results of the first part of the work, carried on in 1936, are set forth below.

The experiments purposed the growing of plants though of same age yet of different sexual maturity and to follow up the changes of growth hormone concentration in them. The following plants were chosen for experiment: millet (*Panicum miciaceum*), hemp (*Cannabis sativa*), sunflower (*Helianthus annuus*), perilla (*Perilla nankinensis*), *Chrysanthemum indicum*, white mustard (*Sinapis alba*) and blew lupine (*Lupinus angustifolius*).

The plants divided into two groups were grown in a green-house in the summer of 1936. Plants of the frist group were given a long (full) day; plants of the second group, a curtailed 10-hour day. Millet, hemp, perilla, chrysanthemum and sunflower «Kubansky 631» rapidly passed the successive stages of development—budding, flowering and seed-formation, when they were given the short day. Under the long (full) day their development slackened considerably, and perilla formed neither buds nor flowers. A quite different picture was observed in mustard and lupine. Treated with long day they soon gave flowers and fruits, while under the short day the mustard produced buds and flowers considerably later and the lupine did not flower at all. Only one strain of sunflower—the sort «Saratovsky ranni»—was not affected by the change in the day length, it budded and flowered at the same time under the long as under the short day.

Thus, in the set of plants taken for experiment we may distinctly discern long day, short day and day-neutral types.

Not only the development but also the height of the plants was affected by the length of day. Irrespective of their response to photoperiodism all the plants grew more rapidly under the long (full) day. In the case of millet, hemp, chrysanthemum and sunflower (sort «Kubansky 631») the difference in the height of the plants grown under short day increased after flowering, whereas in mustard and lupine this difference decreased after they had flowered under the long day. Thus, the effect of the day length on growth was a definite one though different from that exerted on development.

Having determined the concentration of growth hormones in these plants, we should have acquired the knowledge of the trend of these variations, viz. whether they correspond to the development rate or to the growth rate of plants. The determination of growth hormones was performed after a method based on the Went's principle: oat coleoptiles were on one side provided

with agar-blocks containing the hormone of growth. Grown in dark, they were separated from the seed, decapitated 2 mm; the leaflet inside of them was removed, and they were put on wooden nails fixed in a wooden plate. The agar-block with the growth hormone was applied to one side of each coleoptile for $1^1/_2$ to 3 hours, just under the upper cut as to make the growth hormone penetrate into the coleoptile through the uninjured surface. The plates with coleoptiles were brought into moist chambers, on the bottom of which there was poured water, whose level was somewhat above the basis of coleoptiles.

Fig. 1. Growth and development of long day (N) and short day (10) plants of chrysanthemum. Sort «Maria Belaya» (photo taken on Sept. 29, 1936).

Into the agar plate the growth hormone was transferred as follows: the tops cut off the stems were suspended in the moist chamber and agar plates were applied to their upper cuts for 3 hours. Then the plates were cut into 6 equal parts, and the content of growth hormone was determined from the flexure of coleoptiles.

There were the tops which were taken for experiment on account of preliminary tests having shown that the quantity of growth hormones is greater in the top of the plant (see Table 1).

Table 1

Distribution of Growth Hormones Along the Stem of the Plant

Plant	Date of determination	Part of the stem taken for experiment	Flexure in degrees
Lupine	8.VII	Top	33±1.27
		Middle	21±2.31
		Base	12±1.90
Perilla	20.VI	Top	12±1.10
		Middle	9±0.69
		Base	7±1.05
Sunflower	16.VI	Middle	19±1.74
		Base	8±0.90

No growth hormone was discovered in full-grown leaves since the oat coleoptiles gave no flexure under the action of blocks from under the cut-off petioles with leaf blades.

Determinations of growth hormones in the tops of plants showed the long day plants to possess a greater hormone concentration. Part of the respective

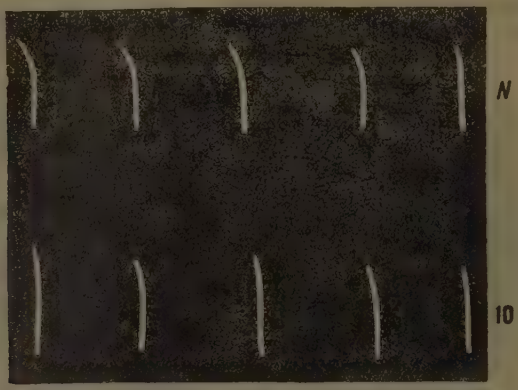

Fig. 2. Flexures of oat coleoptiles due to one-sided supply with chrysanthemum hormone: N—from a long (full) day plant, 10—from a short (10 hours) day plant (photo taken on August 27, 1936).

data are given in Table 2, where the quantity of the growth hormone in mustard, lupine, hemp, chrysanthemum and sunflower is expressed in terms of coleoptile flexure degrees.

The Table 2 shows the concentration of growth hormones to be governed by the same regularities as are observed in the growth of plants; the change of concentration being not connected with the transition of plants from

Table 2

Quantity of Growth Hormones in Plants Treated with Different Day Length

Plant	Date of determination	Day length	Angle of flexure		Period of development
			in degrees	in %	
		I. Long day plants			
White mustard	2.VII	N	16±2.49	100	Flowering
		10	5±0.70	31	Vegetative growth
Lupine	9.VII	N	23±1.60	100	Budding
		10	14±0.0	61	Vegetative growth
		II. Short day plants			
Hemp	13.VIII	N	20±0.57	100	Outset of budding
		10	13±1.45	65	Flowering
Chrysanthemum	1.VII	N	15±1.60	100	Vegetative growth
		10	8±0.40	53	Budding
		III. Day-neutral plants			
Sunflower «Saratovsky ranni»	8.VI	N	20±0.92	100	Budding
		10	11±1.20	55	Budding

vegetative growth to reproductive development. If we compare, for instance, the condition of long and short day plants of chrysanthemum with the content of growth hormones in them (Fig. 1, 2), it will be clear that the increase of growth hormone concentration is connected with a more intensive vegetative growth and formation of vegetative organs and is free from connection with transition of plants to flowering.

We may thus conclude that in all plants, whatever their response to photoperiodism, the formation of growth hormones is going on more intensively when they are given a long day and did not affect the formation of buds and flowers.

Therefore, the part played by growth hormones «auxins» in form-building processes under conditions of photoperiodism is confined to growth processes.

Timiriazev's Institute
of Plant Physiology.
Academy of Sciences of the USSR.

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PFLANZENANATOMIE

EINE NEUE FORM DES EMBRYOSACKES VON *ADOXA*-TYPUS BEI *TULIPA TETRAPHYLLA* UND *T. OSTROVSKIANA*

I. D. ROMANOV

(Vorgelegt von B. M. Keller, Mitglied der Akademie, d. 23. II. 1938)

In der Gattung *Tulipa* waren bisjetzt zwei verschiedene Typen der Embryosackentwicklung bekannt. Treub und Mellink (10) haben bei *T. Gesneriana* den sogenannten «*Lilium*»-Typus der Embryosackentwicklung beschrieben; dieser Typus wurde nachher bei einer ganzen Reihe von Arten der Section *Leiostemones* Boiss. beobachtet (7). Aber die Arbeiten von Bambacioni (1, 2, 3) überzeugen uns dass wir in Wirklichkeit hier einer sehr eigenartigen Form des *Peperomia*-Typus begegnen, welche von dieser Verfasserin zuerst bei *Fritillaria persica* entdeckt wurde. Die Arten der Section *Eriostemones* Boiss. besitzen den Embryosack des *Adoxa*-Typus*, welcher hier durch eine ebenfalls sehr eigenartige Form dargestellt ist (3,4, 7).

Es ist uns gelungen bei *T. tetraphylla* Rgl. und *T. Ostrovskiana* Rgl. einen Embryosack zu finden, welcher als Übergangsform zwischen den beiden genannten Typen bezeichnet werden kann.

Bei *T. tetraphylla* beobachteten wir den Gang der Embryosackentwicklung in folgender Weise (Abb. 1): nach den zwei meiotischen Teilungen entstehen in der Embryosackmutterzelle vier Makrosporenkerne, aber die Makrosporen selbst werden nicht gesondert, da keine Scheidewände entstehen, infolgedessen entwickelt sich aus der Embryosackmutterzelle direkt ein vierkerniger Embryosack (Abb. 1, *a*, *b*, *c*). Gleich nachdem erscheinen im Plasma des Embryosackes einzelne Vakuolen, die augenscheinlich eine wesentliche Rolle bei der Verteilung der Kerne im Embryosack spielen. Sobald die einzelnen Vakuolen zu einer grossen zentralen Vakuole zusammenfliessen, geschieht es meistenteils, dass die Makrosporenkerne in folgende Lage gebracht werden: drei Kerne befinden sich in dem oberen, und ein Kern in dem unteren Teile des Embryosackes (Abb. 1, *d*). Dann treten alle Kerne in dem Embryosacke in die dritte Teilung (Abb. 1, *e*), infolge welcher im oberen Teile des Embryosackes sechs, im unteren—zwei Kerne entstehen. In der Telophase dieser Teilung werden zwischen den Kernen im oberen und im unteren Teile des Embryosackes Zellplatten gebildet. In solcher Weise entstehen im oberen Teile fünf Zellen, unter denen sich eine Eizelle befindet, im unteren Teile entsteht eine Zelle, die einzige Antipode, welche bald abstirbt. Die Zentralzelle des Embryosackes enthält zwei Polkerne (Abb. 1, *f*).

* Unter *Adoxa*-Typus der Embryosackentwicklung verstehen wir den früheren «*Lilium*»-Typus [vergl. Romanov (9)].

Bei *T. Ostrovskiana* findet derselbe Entwicklungsgang des Embryosackes nur in einem Teil der Samenanlagen statt, meistenteils aber geht die Entwicklung bei dieser Art etwas anders vor. Der Unterschied besteht bloss darin, dass die Makrosporenkerne nach der Vakuolisation im letzteren Falle anders, nämlich paarweise verteilt sind, d. h. zwei Makrosporenkerne befinden

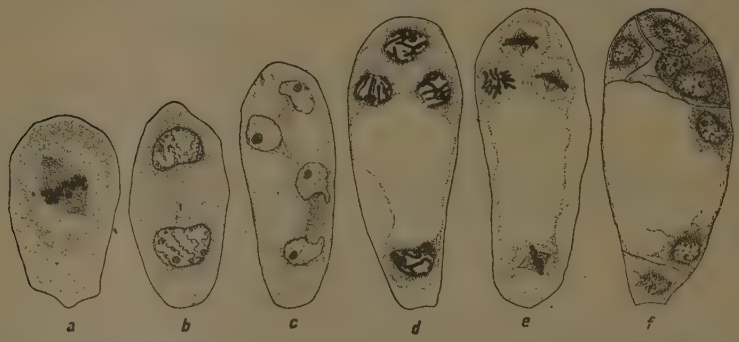

Abb. 1. *Tulipa tetraphylla*. *a*—Metaphase der heterotypischen Teilung in der Embryosackmutterzelle; *b*—Interphase, zweikerniger Embryosack; *c*—der Embryosack nach der homotypischen Teilung; *d*—Prophase des dritten Teilungsschrittes, die Anordnung der Kerne 3+1; *e*—Metaphase der dritten Teilung; *f*—der fertige Embryosack.

sich in dem oberen Teil des Embryosackes, und zwei—in dem unteren Teil; zwischen ihnen die zentrale Vakuole (Abb. 2, *d*). Nach der dritten Teilung (Abb. 2, *e*) entsteht ein Embryosack, welcher einem solchen vom Normal-Typus ganz ähnlich ist: sein Eiapparat besteht aus drei Zellen und im unteren

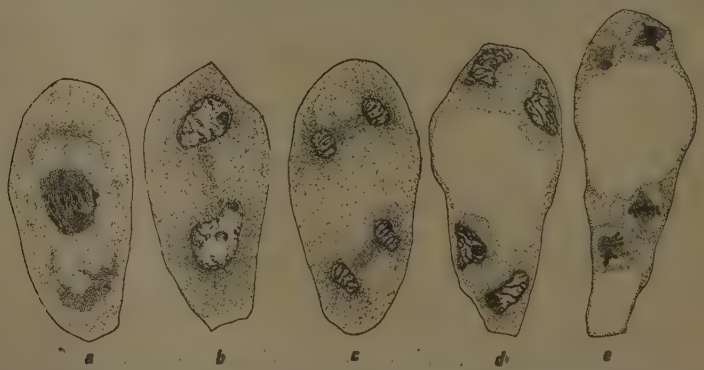

Abb. 2. *Tulipa Ostrovskiana*. *a*—frühe Anaphase der heterotypischen Teilung; *b*—Interphase; *c*—späte Telophase der homotypischen Teilung; *d*—Prophase des dritten Teilungsschrittes, die Anordnung der Kerne 2+2; *e*—Metaphase des dritten Teilungsschrittes.

Teile befinden sich drei bald absterbende Antipoden. Es ist beachtungswert, dass dieselbe Verteilung (d. h. 2+2) der Makrosporenkerne und, selbstverständlich, die Embryosäcke, welche ganz «normal» aussehen, in einzelnen Samenanlagen auch bei *T. tetraphylla* zu beobachten sind. Daraus folgt, dass für *T. tetraphylla* die Verteilung der Makrosporenkerne 3+1 typisch ist, aber ausnahmsweise trift man auch die Verteilung 2 + 2 (es wurden 120 Embryosäcke mit der ersten und 12 mit der zweiten Verteilung beobachtet). Bei *T. Ostrovskiana*, umgekehrt, ist die Verteilung 2 + 2 die

Regel, und bloss als eine Abweichung beobachtet man die Verteilung 3 + 1 (229 Embryosäcke mit der ersten und 37 mit der zweiten Verteilung). [Vergl. Krenke (6), S. 379.]

Die beiden beschriebenen Entwicklungsweisen des Embryosackes sind einander ähnlich, denn in beiden Fällen sind es vier Makrosporen, die einen Embryosack erzeugen, und drei Teilungen, die im Laufe seiner Entwicklung stattfinden.

Nach der Definition, welche in der Embryologie der Angiospermen für den Embryosacktypus angenommen ist (vergl. [8]), gehören die beiden eben beschriebenen Formen des Embryosackes zum *Adoxa*-Typus*, und im diesem Falle muss der Embryosack, welcher typisch für *T. tetraphylla* ist, als eine besondere Form (oder Modifikation nach Palm) des *Adoxa*-Typus gelten, wobei diese Form am nächsten einer anderen Form des *Adoxa*-Typus steht, welche den Tulpen der Section *Eriostemones* Boiss. eigen ist: die vier Makrosporenkerne des Embryosackes befinden sich alle in seinem oberen Teil, welcher, sobald der Embryosack reif ist, 7 (oder 6) Zellen und den oberen Polkern enthält; der untere Polkern und die Antipoden fehlen ganz.

Der Embryosack, welcher für *T. Ostrovskiana* typisch ist, gehört zur klassischen Form des *Adoxa*-Typus (die Verteilung der Makrosporenkerne 2 + 2), welche schon im Jahre 1879 festgestellt wurde [Jönsson (5)], aber für die Gattung *Tulipa* ist diese Form, zweifellos, nur hier zum ersten Mal beschrieben, denn alle früheren Anzeigen des «*Lilium*»-Typus bei *Tulipa* in Wirklichkeit dem *Peperomia*-Typus (*Fritillaria*-Form) gehören.

Dieser letztere wird in der Form, in der er bei *Tulipa* dargestellt ist, durch die Verteilung der Makrosporenkerne 1 + 3 charakterisiert: ein Kern befindet sich, nach der Polarisation des Embryosackes, in seinem oberen Teil, und die drei übrigen Kerne in dem unteren. Darauf folgen noch zwei Teilungen (die dritte und die vierte). Es besteht also in der Gattung *Tulipa* eine ganze Reihe von Kernverteilungen in dem vierkörnigen Embryosack: 4 + 0, 3 + 1, 2 + 2 und 1 + 3. Die ersten drei Glieder der Reihe sind mit dem *Adoxa*-Typus, das letztere mit dem *Peperomia*-Typus verbunden. Das ist ein zweifelloses Zeichen von einer engsten Verwandschaft zwischen diesen beiden Entwicklungstypen.

Es ist ferner beachtungswert, dass auch die letzte geometrisch mögliche Verteilung der vier Makrosporenkerne zwischen zwei Polen des Embryosackes, nämlich 0 + 4, auch manchmal bei *Tulipa* als eine besondere Abweichung stattfinden kann: wir konnten bei *T. Greigii* Rgl. den Embryosack nach der ersten Teilung mit einer Verteilung der Kerne 0 + 2, und Bambacioni—Mezzeti [(3), Abb. 30] bei *T. praecox* nach der dritten Teilung mit einer Verteilung 0 + 8 beobachten. Selbstverständlich entsteht bei solch einer Verteilung der Kerne kein funktionsfähiger Embryosack, und die betreffende Samenanlage bleibt steril.

Biologisches Institut
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ZITIERTE LITERATUR

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* Vergl. die vrg. Bemerkung.

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PLANT ANATOMY

STRUCTURAL PECULIARITIES OF STARCH IN CEREAL AND LEGUMINOUS PLANT SEEDS

By M. I. KNIAGINICHEV and T. M. GORELKINA

(Communicated by N. I. Vavilov, Member of the Academy, 26. II. 1938)

Developmental peculiarities of plants, which belong to different genera, result in different ways of synthesis of the reserve matter in the seed, such as starches and proteins. The colloidal state of these substances and the insufficient knowledge of the physiological condition of the plant beginning to store them make their study difficult. These problems, however, are very actual, for the quantity and the quality of the harvest depend, mainly, on the accumulation and structural peculiarities of starches and proteins present in the seeds.

The present article is a result of an investigation of starches isolated from seeds of different varieties belonging to different species and genera of common cereal and leguminous plants.

The starch was washed away from the seeds with water, and then during 30 min. extracted with 5% solution of potassium sulphate (in the case of cereals) or with 5% sodium chloride (in the case of leguminous plants). Afterwards the starch was again washed with water until no traces of sulphate resp. chloride could be discovered by reaction. To secure rapid drying and purity the starch was twice washed with alcohol and ether; then transferred on to a glass plate it dried there quickly. The content of nitrogen in the preparation obtained was about 0.05%. Comparing starches obtained after the method described with those washed with water only until negative biuret reaction, we found the main properties of starch to have changed but slightly. It is reduced to jelly always at the same temperature; the digestion by diastase of the starch washed with water was lower (19.3 mg and 14.8 mg), the ash content was a little higher (0.22% and 0.39%). Ash and nitrogen content were considerably decreased (from 0.268% to 0.147% and from 0.38% to 0.06% respectively) in starch washed with alcohol and ether after separation with water. These experiments show the starch grain to possess a considerable adsorption capacity and also to change its properties not only under the influence of electrolyte (1) but also other substances.

The study of starches isolated from different varieties of wheat has shown a significant difference in their properties.

Table 1 contains part of data obtained on some species of wheat cultivated in the year 1935 in different districts of the USSR. The jellying tempe-

rature was determined after Samek's method (2); the digestion of starch by diastase is given im mgs of maltose per 1 g of starch, during a 3-hour hydrolysis at pH=6.47 and 30° C.

Table 1

Species and varieties	Pushkin Station of the All-Union Plant Breeding Institute (P. B. I.)				Waluiskaya experimental station (Autonomous Soviet Socialist Republic of Volga Germans)			
	% of ash	% of phosphorous	Jellying temperature	Digestibility of starch	% of ash	% of phosphorous	Jellying temperature	Digestibility of starch
Tr. vulgare Vill. gr. *erythrospermum* Körn. Var. «Novinka»	0.22	0.061	53.5	29.2	0.32	0.092	55.8	25.6
Tr. vulgare Vill. gr. *erythrospermum* Körn. Var. *Erythrospermum* 0341	0.35	0.055	53.8	21.9	0.41	0.082	56.5	40.5
Tr. sphaerococcum Perc. v. *tumidum* from India; after the catalogue of P. B. I. № 23824	0.22	0.047	52.8	36.9	0.24	0.054	54.8	44.2
Tr. durum from Syria; after the catalogue of P. B. I. № 17121	0.28	0.103	51.0	44.2	0.29	0.086	54.0	62.3
Tr. diccocum v. *praecox* from Abyssinia; after the catalogue of P. B. I. № 19258 . .	0.29	0.104	50.8	84.6	0.31	0.095	51.3	84.6

The variability of some properties of wheat starch, isolated from different varieties of two species: *vulgare* and *durum*, has been observed before by other authors (3, 4, 5, 6).

Thus, the influence of environment on the structural pecularities of the starch grain is also expressed in other species of wheat.

The plasticity of the genotype of wheat varieties, as judged by the qualitative characters of the starch grain, is also due to the influence of fertilizers as seen from Table 2 (the steppe station of the All-Union Plant Breeding Institute).

Table 2

Varieties	Fertilizers	% of ash	% of phosphorous	Jellying temperature	Digestibility of starch
Hordeiforme 010	Control	0.268	0.059	52.0	114.0
	Nitrogen and phosphorous	0.207	0.058	51.0	168.5
Lutescens 062	Control	0.135	0.096	52.0	101.0
	Nitrogen and phosphorous	0.215	0.128	50.5	133.3

Thus, the ability to synthesize starch grains of definite structure is different in different varieties and depends not only upon the genotype of plant but also on external factors.

Other genera of cereal and leguminous plants are also able to synthesize starch, whose properties are different, according to the variety. However, when the structure of the starch grain is destroyed by jellying, we can observe slight differences both within some genera and between separate varieties. See Table 3, which contains a number of data obtained.

Table 3

Name of the culture	Locality	Digestibility of starch by diastase	
		Structural starch	Jellied starch
Tr. vulgare gr. *graecum* var. *Graecum* 0283	Moscow	6.6	611.3
Ditto	Dnepropetrovsk	29.2	602.0
Tr. durum gr. *hordeiforme* var. *Hordeiforme* 0189	Moscow	25.6	601.7
Ditto	Dnepropetrovsk	70.1	611.0
Tr. monococcum gr. *flavescens;* after the catalogue of P. B. I. № 20631	Pushkin station of P. B. I.	58.6	644.4
Ditto	Dnepropetrovsk	29.2	619.6
Secale cereale var. Vjatka	Pushkin station of P. B. I.	25.2	666.8
Ditto, var. Elissejewskaja № 7838	Ditto	21.9	674.8
Hordeum vulgare distichum var. *persicum* 064—sommer sowing	Burenski station	70.1	648.0
Ditto—winter soving	Ditto	77.3	683.1
Hordeum vulgare tetrastichum v. *pallidum* 043	Burenski station	84.6	648.0
Phaseolus vulgaris v. *Robuste*	Steppe station of P. B. I.	6.6	694.0
Ph. acutifolius v. *latifolius* Treeman v. *Tepary* № 7966	Ditto	36.9	676.0
Vigna sinensis from Egypt (after the catalogue of P. B. I. № 1)	Ditto	18.3	695.0
Pisum sativum ssp. *Sativum* v. *Victoria* Strube	Pushkin station of P. B. I.	8.4	624.7
Pisum sativum ssp. *Sativum* v. *Sacharny mozgowoy*	Ditto	18.3	540.9

Other species and varieties of these genera have shown analogous results.

The data of digestion of barley structural starch by diastase are of interest. The digestion is rather high and even. As to jellied starch, its digestibility in barley differs but very little from that in other cultures.

It is well known that the size of starch grains varies within a wide range. This allows to separate the starch into fractions according to size. Data in Table 4 show the digestibility of different fractions of starch from different varieties cultivated at Dnepropetrovsk.

From Table 4 we see that the digestibility of structural starch varies greatly with the size of granules whereas the jellied starch shows only slight differences. In this connection we have to point out the works of anatomists([7],[8]) which have shown that the large granules are formed in the first stage of ripening («green maturity») from the plastids while the small ones are formed in the next stage from the chondriosomes. The hydrolysis of small granules pro-

Table 4

Name of the culture	Size of granules	Digestibility by diastase	
		Structural starch	Jellied starch
Triticum vulgare gr. *graecum* var. *Graecum* 0283	Large	22.6	618.3
	Small	40.3	609.1
Tr. monococcum gr. *flavescens;* after the catalogue of P. B. I. № 20631	Large	24.2	630.2
	Small	46.3	632.4
Secale cereale var. Elissejewskaja; after the catalogue of P. B. I. №. 7838	Large	21.9	674.0
	Small	55.0	644.0

ceed better on account of the greater surface subject to the action of diastase and owing to their structural peculiarities. The rate of hydrolysis of structural starch by 10% hydrochloric acid is, in some cases, greater than by diastase (3 hours at 30°). The jellied starch, on the contrary, is by far less susceptible to the action of hydrochloric acid than to that of diastase (Table 5).

Table 5

Name of the culture	Locality	The content of maltose in mg per 1 g of starch			
		Hydrolysis by diastase		Hydrolysis by 10% hydrochloric acid	
		structural starch	jellied starch	structural starch	jellied starch
Triticum vulgare gr. *graecum* var. *Graecum* 0283	Moscow	6.6	611.3	29.6	38.8
Triticum durum gr. *hordeiforme* var. *Hordeiforme* 0189	Moscow	25.6	601.7	21.2	36.6
Phaseolus vulgaris var. *Robuste*	Steppe station	6.6	694.0	34.4	56.4
Vigna sinensis (after the catalogue of P. B. I. № 1) from Egypt	Steppe station	18.3	695.0	42.4	48.8

Other species and varieties of these genera have shown analogous results.

The weaker hydrolysis of the jellied starch by hydrochloric acid shows that there remain some structural links in the paste, which are destroyed by diastase much more quickly than by hydrochloric acid. Several authors ([9], [10], [11], [12]) have observed the differences of starch properties in different varieties of some crops.

At present, the starch formula is expressed as follows :

$$[(C_6H_{10}O_5)_x]_y,$$

where x denotes the polymerization coefficient and y—the association coefficient.

As the photosynthesis is connected with chlorophyll and its first products are sugars, the majority of genera seem to form the initial products of starch according to one scheme. The differences according to genera, species and varieties appear only in subsequent stages when the polymeriza-

tion of glucose residues and their association take place. From this point of view it is possible to explain the absence of differences between varieties, species and even genera in the case of jellied starch hydrolysis.

The above data allow us to draw the following conclusions.

Varieties, species and genera of plants form starch granules differing in their properties (jellying temperature, hydrolysis by diastase and hydrochloric acid, ash and phosphorous content).

When hydrolysis is made by diastase, the sugar formation in jellied starch is almost the same within the genera investigated: wheat, rye, barley, haricot bean, cow-pea (*Vigna sinensis*) and pea (*Pisum sativum*).

The structural peculiarities of the starches depend not only on the variety, species or genera but also on the climate and agricultural practice.

Obviously, the initial products of the starch synthesis in the plants investigated are closely related and only in subsequent periods of development, with the outset of association of starch links, there arise differences according to genera, species and varieties of plants.

Biochemical Laboratory
of the All-Union Plant
Breeding Institute.

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ZOOLOGY

CONTRIBUTION TO THE ECOLOGY AND MORPHOLOGY OF *CALANUS TONSUS* BRADY (=*CALANUS PLUMCHRUS* MARUKAWA) OF FAR-EASTERN SEAS

By K. A. BRODSKY

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The systematic composition of the zooplankton of the northern part of the Pacific and the adjoining marginal seas has at yet been insufficiently studied. The *Copepoda* are scarcely studied at all. For this reason, the solution of the oceanographic problems, depending upon the knowledge of the composition of the plankton, is extremely difficult. In this respect, as well as from the viewpoint that the *Copepoda* play a considerable part both in the sea economy and as indices of the hydrological conditions, all the data showing the exact composition of the *Copepoda* in far-estern seas are of great importance.

In the present note the following points in the biology of the *Calanus* are taken into consideration.

1. The seasonal variability in one species of the genus *Calanus* is manifested in the existence of two forms, with some different morphological and ecological characters; these two forms being established on the basis of a morphological and ecological study of the species *Calanus tonsus*. What the author believes to be forms has been described as two independent species (*Calanus tonsus* Brady and *Calanus plumchrus* Marukawa).

2. The second point which has to be taken into consideration, is very important for the knowledge of the biology of *Copepoda*. This is the existence of two phases in the V copepod stage of *Calanus tonsus*, which has been first recorded by Campbell (4) and is in full agreement with author's analysis of the material available. The above-mentioned peculiarities in the ecology of *Calanus tonsus* are of general importance for the study of the *Copepoda* of the far-eastern seas and the northern part of the Pacific, since a great number of the *Copepoda* of the aquatorium under investigation are di- and polymorphous, and a study of the ecology of a species, without considering the presence of individual races and ecotypes, frequently gives too vague results, which make it impossible to use the given organism as an index of the hydrological conditions or of the character of certain regions of the sea. Such polymorphous species are: *Pseudocalanus elongatus* Breck, *Paracalanus parvus* Claus., *Calanus Finmarchicus* Gunner and a number of other *Copepoda*, but especially the first and the last of the species mentioned. Doubtless the *Calanus Finmarchicus* of the Japanese Sea, the Californian Bay etc. is a different form, perhaps even species of the *Calanus Finmarchicus* of the English channel or the Barentz Sea.

We shall turn now to the material on *Calanus tonsus*.

The female *Calanus tonsus* was described by Brady in 1883 (1). The male *Calanus tonsus* has been described later by Campbell (2). In 1921 Marukawa described a new species of the genus *Calanus Calanus plumchrus* (3), according to the V stage.

When studying the material obtained from the Japanese Sea, from certain regions of the northern part of the Pacific, from the Sea of Okhotsk and the Bering Sea*, the author has found there two forms of the *tonsus*, both with longer and shorter II bristles, considering from the inner margin of the caudal branches. In comparing the two forms the author has been able to note only the following differences, viz: the II bristle, considering from the inner margin on the caudal branches of the form *plumchrus***, is more than twice as long as the same of a typical form *tonsus*. The feathering of these bristles is also different. A comparison of the swimming feet, the mouth parts, the dimensions and the shape of the body of the two forms of *Calanus tonsus* under description, showed no differences. The length of the body for *C. plumchrus* (V stage) is according to Marukawa 4.2—5.2 mm; that of the *C. tonsus* (V copepod stage) is according to Campbell 4.5—5.0 mm. Consequently we have to do here with two forms of the same species *Calanus*, distinguished from each other only by the structure and length of the II bristle on the caudal branches. We refer these two forms to the same species *tonsus*, because of: 1) the complete similarity of the habitus, except the structure of one bristle on the caudal branches only; 2) in the test-portion taken in May from the Japanese Sea a large number of species of *C. tonsus* at the V copepod stage has been discovered. The comparison of the specimens of this species showed the presence of specimens differring in nothing but long or short bristles. Specimens with bristles of a length ranging between the two extreme forms have also been observed.

From the distribution and the ecology of these two forms the author supposes them to be nothing else, but two different generations of the same species. Their morphological difference, manifested in the different length and feathering of the caudal bristles, may depend on temperature conditions.

The distribution of *Calanus tonsus* extends over wide areas. This species is found in the southern hemisphere: on the shores of Australia, New Zealand, South America; it has been observed in the Atlantic ocean and on the North-American Coast of the Pacific (Vancouver, S.-Diego, Queen Charlotte islands); in the Bering, Okhotsk, Japanese Seas. The presence of generations with long and long-feathered caudal bristles (*f. plumchrus*) is established beyond doubt for the three far-eastern seas (Japanese, Okhotsk, Bering).

The *Calanus tonsus* (both *f. typica* and *f. plumchrus*) is a common species playing a considerable part in the nutriment of the fishes (herrings) feeding on plankton and in the sea economy. With regard to George's Strait this problem has been discussed in a paper by Campbell (5). This species is also found in large numbers in the Okhotsk Sea and in the northern part of the Pacific. In the Japanese Sea it is also observed in large numbers during certain seasons. According to the author's data, the seasonal distribution of this species may be described as follows: In May, in the surface layers *Calanus tonsus* is met with in large numbers in the layers of 0 to 25 m and 25—50 m; in the layer of 50—100 m the number of this species drops. At this time all the stages from I to V may be found; adult males and females are not recorded. The V and IV stages are the most abundant, but occasionally the III

* It would be interesting to note, that the specimens of this species (both the *typica* and the *plumchrus* forms) from the Bering Sea were larger in size than similar specimens from the Japanese and Okhotsk Seas and from the Pacific.

** With a length of the body=4.6 mm, the length of the bristle=2.8 mm.

stage prevails. The I stage is the least frequently observed. Such are the conditions throughout May and June. As soon as the water becomes warm, this species descends to deeper layers. During this period almost exclusively *f. plumchrus* is found which in living state is easily discernible by the nacked eye, owing to its long and bright-coloured furcalar bristles.

Throughout the summer months *Calanus tonsus* is absent from the upper layers, while it is met with in great numbers below 100 m and chiefly in the form of the V copepod stage. During the same period adult male and female *C. tonsus* are found in small numbers. With temperature decrease in the upper layer of water this species begins to ascend, adult males and females being frequently found in upper layers. In December females are prevalent. The cycle of development is completed in one year; the propagation takes place at a considerable depth; in summer it takes place on a small scale, during the cold seasons it is more intensive in layers below 100—150 m. By the end of May, June, this *Copepoda* passes through the nauplius and the early copepod stages and descends to the depth in the form of the V copepod stage, until it attains the VI copepod stage, i. e. that of adult males and females. In summer a certain approach of the V stage to the near-surface layers may be observed at night, since, according to the author's observations, this species possesses a rather clearly pronounced diurnal vertical migration. But during the day, in August for instance, with a depth of 200 m, the greatest accumulation of this species has been observed in the layer from 100 to 200 m; with greater depths they descend deeper. Contrary to the conditions observed in May, in August the author became convinced of the presence of *f. typica*. *F. typica* and *f. plumchrus* are encountered in different numbers, according to season and depth, namely, the generations which at spring-time during the V copepod stage ascend nearer to the sea surface, refer to *f. plumchrus*; generations spending the warm seasons at considerable depths (in the form of the V stage) should be referred to *f. typica*. This presumption is also supported by the fact that in May the specimens of the IV copepod stage, in the presence of a mass of the V stage of *f. plumchrus*, did not possess long bristles. In the Okhotsk Sea the conditions are somewhat different, since the IV stage with long bristles has been observed alongside of the V stage of *f. plumchrus*. The dimorphism may possibly be manifested there somewhat earlier than in the V stage, but in the Japanese Sea this has not been observed. In the Japanese Sea the temperature of water, for the time of predominance of generations with long and long-feathered caudal bristles (*f. plumchrus*), in May and June is about 5—9° in the near-surface layers. In winter-time and in summer at a great depth (over 150 m), when almost exclusively *f. tonsus* is encountered, the temperature of water is not above 1° C. It may be assumed that the elongated caudal bristle feathered by long and broad bristles of the *f. plumchrus* is of adaptative nature. In examining the structure of the caudal bristles of *Copepoda*, we find in most cases an analogous feathering for the forms found in warm waters and at the surface (e. g. the species of the genus *Calocalanus*, especially *Calocalanus plumulosus* Claus.); among the deepwater forms a considerable number of species is distinguished with a strongly elongated caudal bristle, but the latter mostly nacked, devoid of feathering.

The existence of two forms of *Calanus tonsus* in the V stage is very interesting from the biological point of view. The stage of sexual maturity of the *Calanus tonsus* is very short, similar to the *Calanus cristatus* ([6]), and owing to the reduction of the masticatory apparatus males and females are unable to feed. Owing to this nutritious substances are accumulating during the V stage, which is the longest as compared with other stages [this has been established by Campbell ([4]) and wholly supported by the author's observations]. Due to such an accumulation of reserves, the shape of the body itself and the course of the V stage of *Calanus tonsus* is changed.

As the time of moulting approaches, the specimens of the V stage become less transparent, and finally completely opaque, getting gradually thicker at the same time. The line of the back is arching and even the outlines of the frontlet are somewhat altered, becoming near the vertical. At this time these *Copepoda*, looking thoroughly swollen, contain large quantities of fat (Fig. 1, *b*). Thick and opaque specimens have been found in the Japanese Sea at depths over 500 m, i. e. just in those horizons, where the transformation of the V stage to the adult stage takes place in summer. Such an alteration of the form of the V stage seems to make it possible to speak of the existence of two phases, namely the transparent phase, with a small quantity of stored reserves of nutritious substances, with a strait or even concave line of the back (Fig. 1, *a*), and the opaque one, directly preceding moulting, with a convex back. Such phases may, undoubtedly, be observed in all *Copepoda*, whose males and females are unable to feed, owing to a reduction of the masticatory apparatus.

Analogous peculiarities are observed in different groups altogether, viz. in insects (*Ephemeroptera*, certain *Diptera*, etc.) whose adult stage lives for a short time and this at the expense of the nutritious substances stored during the preceding stages.

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EXPERIMENTAL ZOOLOGY

MORPHOGENESIS FOLLOWING THE INSERTION OF THE PARTS OF VARIOUS ORGANS UNDER THE SKIN OF THE AXOLOTL

By N. V. NASSONOV, Member of the Academy

I. LUNG AS ORGANIZER

In preceding articles ([1–3]) we discussed the possibilities of obtaining new formations by inserting under the skin of adult axolots, in specific parts of their body, certain tissues, such as epithelial, osseous, cartilaginous and muscular ones. As in the majority of cases it is impossible to educe sufficiently separate tissues from the organs, we utilized for our further experiments the complexes of tissues contained in various organs and we inserted them under the skin in small bits 0.3—0.4 mm long and wide. We have made over 5000 homoplastic and heteroplastic insertions of the following organs: lung, gills, small intestine, liver, bladder, kidneys, brain, eyes, heart, fins and sexual glands. I have succeeded in obtaining additional organs in various stages of development, by combining, in some form or another, the homotopical, heterotopical and heteroplastic insertions of the tissues of all above mentioned organs, except the sexual glands. The experiments are being carried on. For heteroplastic insertions the bits of tissues were taken from fishes (crucian and sterlet), tritons (*T. cristatus* et *T. taeniatus*), frogs (*R. esculenta* et *arvalis*), lizards (*L. agilis* et *Agama vivipara* species) and white rats.

In the present article we intend to give a brief description of the experiments concerning the insertions of the parts of lung, small intestine and gills as organizers.

In order to establish the presence of the organizing substances in the internal organs, we introduced, first of all, under the skin of the axolotls the bits of their lungs. In the course of 1935 we made 40 homoplastical insertions of the walls of a normal lung of an adult axolotl. We obtained clear indications of the appearance of new formations in the form of additional parts of the leg. I have already mentioned this in my article «Influence of Various Factors on Morphogenesis Following Homotopical Subcutaneous Insertions of Cartilage in the Axolotl» [([2]), p. 47]. To make it more sure I undertook a second series of 40 insertions of the bits of the same organ, and I have obtained the following results.

Shortly after the insertions have been made, there appeared externally the tumours of a more or less considerable size. In some cases we observed inflammatory processes which manifested themselves externally in the form of the reddening of the skin at the summit of the tumour. When we remarked the beginning of the appearance of new formations they looked like whitish or grey tubercles appearing on the tumours 40 or 50 days after the insertion of the bits of lung. On the whole, we have observed 17 new forma-

tions*. Nine of them were considerably developed and looked like the additional parts of the legs. In eight cases the development stopped at an early stage, and the formations looked like buds or sharply isolated tubercles filled up within by the connective tissue. These formations were very similar to the embryos of the legs at the time of their appearance during the development. I have already described these formations as developing sometimes after the insertion of the cartilage under the skin of the leg [(2), p. 99] and I have called them primitive formations**.

One of the most developed additional formations reached 2 cm of length and appeared at the base of the tibia (Fig. 1, *a*) on a tumour 0.8 cm long and 0.5 cm wide (Fig. 1, *c*). It looked like fully developed tibia and foot with five digits. In its structure this formation appears to be a duplicate of the part of the leg of the host extending from the base of the tibia, from where it evolves, to the summit of the leg, but being only of a smaller size. As I have already mentioned previously, the maximal degree of development of the duplicating or, otherwise, of the additional part of the leg does not generally exceed the limit which would be attained by the regenerate, provided that the latter evolves after an amputation on the spot of the appearance of this additional part of the leg [(4), p. 202]. In other words, the additional (duplicated) formations of the parts of the extremity are duplicating, to some or another extent, the part of the host's leg, beginning from the spot where the additional formation evolves; these additional formations do not contain the elements of the part of the extremity which are present in the host proximally from the place from which this formation evolves.

Fig. 1. The left hind leg of an axolotl with full additional formations obtained on the tibia after the insertion under its skin of a bit of tissues of the walls of the lung of the axolotl. *a*—pentadactyl additional formation; *b*—undeveloped tridactyl formation; *c*—tumour.

At the beginning of the development there appears in the front of the tumour a lightly coloured spot, this being the consequence of the decomposition of the cutis. Then a white semi-transparent tubercle is formed which, similarly to the embryo of the aforesaid additional part of the extremity, is increasing in length and then is differentiating into tibia and pentadactyl foot. During the development of the additional part of the extremity described above on the tumour there appears another embryo of the additional extremity which fails to develop and remains in the shape of an abnormal tridactyl appendage (Fig. 1, *b*) situated at the enlarged base of the additional pendactyl formation. It is possible that in this case two insertions were made incidentally too closely one to another; therefore, the tumours fused and the embryos of the additional formations, which appeared on them, approached one another, while the development of one of them became prevalent.

Besides this, we have obtained two tridactyl additional appendages. One of them was also formed on the tibia, but it did not attain the stage of development which was possible for the additional formation similar to that des-

* The high percentage of negative results is explained by the fact that all conditions favourable for the development of new formations are not yet sufficiently ascertained.

** Such formations appear after the subcutaneous insertion of desiccated cartilage. Of late I succeeded in obtaining them by inserting under the skin the cartilage which was heated to the temperature of 98°.

cribed above. It had 0.5 cm in length and the base of its fully developed skeleton consisted of a single transversal cartilage (Fig. 2, *a*) corresponding to the cartilages of the tarsus. This transversal cartilage joined, in its turn, a long cartilage (Fig. 2), probably corresponding to tibia and fibula, as well as the metatarsal cartilages situated distally. Each digit had differentiated cartilages—their phalanxes (Fig. 2) with muscles attached to them (*m*).

Fig. 2. A longitudinal section through the tridactyl additional formation on the left tibia cutting the two digits and their base. *a*—phalanxes of the digits, *b*—basic transversal cartilage, *c*—proximal cartilage, *m*—muscular fibres.

Another incompletely developed formation which was also tridactyl was obtained on the autopody and reached 0.4 cm in length (Fig. 3). In the skeleton of its digits it possessed differentiated cartilages joined to one basic cartilage probably corresponding to the fused metatarsal and tarsal cartilages.

Then we have obtained one formation in the shape of a small protuberance with an appendage which was bifurcated on the summit. One of the branches disappeared with time and this formation assumed the shape of a digit with a number of cartilages inside. Besides this, six digital appendages about 0.4 cm long and containing inside cartilages were also formed. Two of them approached one another, being both situated on a large tumour (Fig. 3, *a*). On the same tumour, near them, there was a single appendage which had the form of a digit 0.7 cm long.

Fig. 3. The left hind leg of the axolotl with the additional formation *a* in the shape of a tridactyl appendage obtained on the foot after the insertion under its skin of a bit of the lung of the axolotl.

The sections of the tumours at the places of insertions have given the following results. On the sections made 18 days after the insertion of the bits of lung there were observed accumulations of phagocytes around the insertions and in their normal cavities. Partially the phagocytes penetrated into the thick of their walls and then the epithelium covering the cavities of the walls of the lungs was clearly seen. The cutis was not destroyed. Sometimes the process of the destruction of the insertion by the phagocytes can last for a very long period. An incomplete destruction was observed on the 42nd and even on the 170th day, but sometimes the process can become considerably intensified on the 30th day. In this case the structure of the walls of the lung was greatly decomposed and there were only seen the fibres of the connective tissue and the remnants of

the cells of lung tissue situated between them, as well as phagocytes which penetrated also into the cutis. Toward the 40th day the picture remains unchanged, and the inserted bits of the lung are probably totally destroyed. But it is more frequent that about 40—50 days after the insertions we observe quite a different picture, namely, in the thick of the insertions there appear cavities which are continually increasing. The tissues of the walls of the cavities change considerably and, evidently, their histolysis takes place. Sometimes the cavities attain very considerable dimensions and they are often situated over the insertion under the epithelium. The cavities are filled with a fluid which is probably accumulating there as a product of histolysis. Sometimes the walls of these cavities are very thin and

Fig. 4. The section through the rudimentary formations of the cartilage *a* and of the appendage *b* in the form of a protuberance on the skin with cartilage inside it (*c*); *d* and *d'*—insertions of two bits of the tissues of the walls of the lung which has considerably changed, probably as a result of histolysis; *e*—the cavity filled with fluid which coagulated owing to the treatment by the reagents; *f*—cutis; *m*—muscular fibres.

they consist chiefly of fibres and remnants of the nuclei of the cells of the insertion. At that time there appears the accumulation of cells from which there is formed the embryo of the additional extremity and which lies nearer to the epithelium. The cutis over the places of the insertions is decomposed more or less on quite a considerable surface. With respect to new formations, only cartilages are sometimes formed (Fig. 4, *a*). Sometimes a protuberance of the epithelium is formed over them, and the cartilage produces an appendage inside it (Fig. 4, *c*); as a result of this there appears an appendage in the form of an undeveloped digit (Fig. 4, *b*). Sometimes the cartilage develops directly at the base of the appendage.

There is no doubt that in this case the development of the cartilage goes on in the subcutaneous connective tissue, for the former can develop quite independently of the perichondrium of the host being separated from the latter by the layers of muscular fibres. However, the cartilage can also develop from the perichondrium of the host, if the organizer is in direct touch or in close neighbourhood with the perichondrium. Sometimes the new formation proceeds further and the appendage develops, as we have already seen, into a di-, tri- or pentadactyl part of the extremity with differentiated cartilages. Simultaneously with the disarticulation of the skeleton there goes on, in certain cases, the formation of muscular fibres.

Thus, we see that the subcutaneous insertion into the leg of an organizer in the form of lung walls, as well as of cartilage, is able to produce variously undeveloped formations, as it is observed in the case of subregeneration. The cause of this phenomenon remains unascertained. It is probable that in the

case of undevelopment some organizing substances fail to become active. The final solution of this problem depends mainly on biochemical investigations along this line which are actually carried on by my collaborator A. B. Silaev under the supervision of N. D. Zelinsky, Member of the Academy. These investigations have not yet been finished, but they proved already that the insertion of the products of an alkali histolysis is able to produce digital appendages with a cartilage within them. This fact demonstrates clearly that in this case we deal with organizing substances, among which we find the supposed stimulators of proliferation of the epithelium and the regulators of this proliferation which are producing the appearance of the protuberances of the epithelium on the surface of the skin ([2–3]), as well as of hondrogen*. The investigations are being carried on.

On the basis of the above we can conclude that the tissues of the lung contain approximatively in the same degree the organizing substances and the cartilage. We can conclude further that these substances are also active only when being placed into a definite medium, in this particular case—into the medium surrounding the tissues of the lung under the skin of the leg. Apparently in this case we observe a phenomenon of the same kind as that which is caused by the insertion of a muscular fibre under the entoderm of an amphibian gastrula and which, according to the observation of Woerdeman ([6]), produced an embryo of a nervous system, while, according to my own observations, such fibres produced no formation whatever under the skin of the leg ([1]). Probably in the lung tissues, as well as in the cartilage and in the muscles, some or another organizing substances are combined with certain substances which enter into the composition of the tissues of the lung and are unable to manifest their activity, but, being placed into the medium under the skin of the legs, they are liberated from this combination and become active.

Fig. 5. New formations (*a* and *b*) obtained by inserting under the skin of the tail of an axolotl a bit of its lung.

40 homoplastic insertions into the leg of the axolotl of the bits of boiled lung and 40 similar insertions of the bits of lung dried in room temperature have given negative results. 8 insertions of the bits of the normal lung into the head of an axolotl and 40 similar insertions into its side, between the bases of the legs, have also failed to give any formation.

We have also made 40 insertions of the bits of lung walls into the back of an axolotl and 40 similar insertions into its tail. On the back, as well as after the insertion of the cartilage ([3]), no formations were obtained in any of these cases. After the insertions into the tail these formations appeared only in 3 cases. One of them looked like a semi-transparent tubercle, the second like a scale 1 mm high, and the third like a cylindrical appendage of the same length (Fig. 5, *a* and *b*). They have failed to take the form of a tail-like appendage which was obtained by the insertions of cartilage into the tail; the first of them deteriorated after a certain time.

Thus, as we see, the organizing substances of the lung tissues, when being placed under the skin of the tail, manifest their form-building capacity in an inferior degree than when being placed under the skin of the leg. The medium surrounding these substances in the tail, in its influence on them, probably differs from the medium of the legs. With the change of the medium chemical changes are probably taking place in the inserted tissues of the lung

* I call hondrogen the substance which is contained in the water extract of cartilages. The in vitro culture of the connective tissue in the plasm of the blood of an axolotl with the addition of the aforesaid extract is able to transform this tissue into a cartilage [Nassonov, 1934 ([7])].

which are able to determine only the degree of the activity of the organizing substances contained in these tissues, or are able to make active only some of them.

The sections have demonstrated that some bits inserted into the tail have not changed, and the living tissue of the lung can be easily seen in the sections made 140, 230 and even 330 days after the insertions under the skin have been made. On this occasion we can also observe that the bits of the living tissue are inosculated with the connective tissue surrounding them and that the capillars of the vascular system of the host have penetrated into the inserted tissue. I have had already the occasion to describe a similar phenomenon in the case of the insertion of entire cartilages of the axolotl under the skin of its leg ([5]). The same phenomenon was observed by me after the subcutaneous insertion into the leg of an axolotl of the bits of its liver, when the fusion of the vascular system of the host with the capillars of the liver was very clearly expressed both in the inflow and outflow blood vessels of the inserted bit of liver. In all these cases we did not obtain any formation. In this we see a clear indication to the effect that in order to obtain a new formation the living tissue inserted under the skin must undergo some changes, probably of a chemical character.

As we have already seen, there are grounds to suppose that, in the case of new formations in the tissues of the lungs inserted under the skin, histological processes are also going on. Further biochemical investigations will show to what an extent their products are able to influence the development of additional formations.

40 homoplastic insertions of the bits of a boiled lung into the leg of an axolotl and 40 similar insertions of the lung dried in room temperature have given negative results. With regard to the heteroplastic insertions similar results were obtained after 32 insertions of the bits of the lung of a triton (*T. cristatus*) into the leg of an axolotl. On the place of 55 insertions of the bits of the lung of a frog into the leg of an axolotl we have obtained in two cases small (about 1 mm high) conical appendages on the autopodies. The sections have shown that inside these formations, in their connective tissue, there are present the oblong non-differentiated cartilages.

40 insertions of the bits of the lung of a lizard (*L. agilis*) into the leg of an axolotl produced no formations. We were even unable to see any stimulation of the proliferation of the epithelium which we observe after the insertions of the cartilage of a lizard into the leg of an axolotl.

We see that all these heteroplastic insertions of a normal lung into the leg of the axolotl probably contain the organizing substances in a smaller number and in an inferior degree than in the case of the corresponding insertions of cartilage. Among other heteroplastic insertions, we have made 50 insertions into the leg of the axolotl of the lung of a white rat which did not produce any new formation similarly as the corresponding insertions of the cartilage of a rat([2]).

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EXPERIMENTAL ZOOLOGY

MORPHOGENESIS FOLLOWING THE INSERTION OF THE PARTS OF VARIOUS ORGANS UNDER THE SKIN OF THE AXOLOTL

By N. V. NASSONOV, Member of the Academy

II, SMALL INTESTINE AS ORGANIZER

In order to establish the importance of the digestive organs as organizers we have chosen, first of all, for our experiments the middle region of the small intestine, an organ of endodermic origin. Firstly we have made a series of 40 homoplastic insertions of the bits of the wall of a normal small intestine under the skin of the legs of an axolotl, but, as the obtained results were vague, we have made another series of 30 similar insertions. It appeared that in the cases when the bits of the small intestine were placed under the skin somewhat retreating from the edge of the wound caused by the incision of the skin through which the insertion had been made, we never obtained additional formations like those observed after the insertion of cartilage or lung. The sections have shown that at the places of insertions the cutis remained undamaged or only partly damaged, while the colagen fibres remained always undamaged. We observe evidently in this case the same phenomenon which takes place when the skin is covering the wound after the amputation of the leg. It is known that under such conditions there occurs no regeneration. The similarity between regeneration, on the one hand, and the development of the additional formations after the insertion of some tissues, on the other hand, is complete.

However, when the tissues of the small intestine were placed under the skin of the leg close to the edge of the wound on which the cutis did not develop for a certain time and which at the beginning was covered only by epithelium growing from the surface of the skin, at that time the processes were going on under the epithelium and these processes produced new formations at the edge of the wound. The tissues of the small intestine, as contrary to cartilage and the tissues of the lung, were unable to decompose the cutis and to give the possibility of development of additional formations. This can be explained by the fact that the coriocide* contained in the walls of the small intestine is combined with another substance and cannot manifest its activity similarly to the substances which stimulate the propagation of the epithelium and which are present in the muscles of the axolotl (Nassonov, 1937, III, p. 385). This combination remained unchanged also after the

* For the sake of shortness I have given the name «coriocide» to the substance which, as I suppose, is contained in certain tissues and the action of which on the skin consists in the destruction of the connective tissue.

insertion of bits of the small intestine under the skin of the leg. These considerations are confirmed by the results of the heterotopic insertions of bits of the small intestine walls when the coriocide, being placed in a different medium, as we shall see in the sequel, is manifesting its activity.

The new formations mentioned above were in the majority of cases of a primitive character, namely, they had the shape of small cupola-like or conical protuberances about 0.1 cm high and situated in various parts of the leg. The cartilage, as was shown by the sections, did not develop inside. We obtained only two better developed new formations. One developed on the foot and looked like a digit with an enlarged base situated at the edge of the wound, where there was a tumour on the skin. This new formation developed approximatively within the period of a month and was about 0.15 cm in height. After three months its degradation began and on the 170th day, as shown by the sections, it had inside no cartilage skeleton. The second new formation developed on the knee (Fig. 1) and reached the length of 0.4 cm*. It had the form of an appendage with two digits. Inside of each digit there was one cartilage undivided into phalanxes. The cartilage was investigated on the 170th day and it had an ossification on the summit of the digit (Fig. 2,*a*). The muscles did not develop. I obtained more developed formation in the case of 20 homoplastic insertions of the bits of the small intestine, the structure of which was considerably destroyed by grinding between glasses. I obtained two formations evolving from under the edge of the wound through which the insertions had been made. One of them looked like a small appendage on the femur 0.2 cm long with two cartilages undivided into phalanxes, while the other looked like an appendage 0.6 cm long and possessed four digits, two of them being inosculated one with another (Fig. 3, *a*). They all contained cartilages divided into phalanxes to which muscular fibres were attached; besides this, at the base of the appendage there were cartilages corresponding probably to the carpal and metacarpal cartilages.

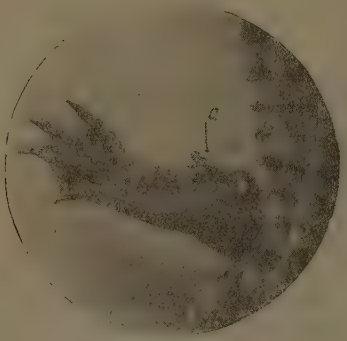

Fig. 1. The left hind leg of a black axolotl with didactyl additional formation *a* which evolved from under the edge of the wound on the knee after the insertion of a bit of the normal small intestine under the skin of the leg.

Thus, we have to assume that in the walls of the small intestine there is contained hondrogen stimulating the proliferation of the cells of the epithelim and of the connective and the muscular tissues, as well as that there appear the regulators of the proliferation of these cells.

40 homoplastic insertions under the skin of the leg of an axolotl of boiled bits of the small intestine, as well as 40 similar insertions of dried bits produced no new formations. 24 homoplastic insertions of the bits of the small intestine into the head of the axolotl and 40 similar insertions

* As the beginning of the regenerative processes, as well as of the processes of development of additional formations, consists in the fact that the cells of epithelium, of muscles and of the connective tissue begin to proliferate intensively in specific places, so the cutis and the corium, which do not allow such propagation, are probably preserving the organism from the development on the body of various abnormal new formations in the shape of tumours. We shall deal more in detail with this problem in one of the following articles.

into its back failed to produce any new formation, as it was the case with the insertions of cartilages. 40 homoplastic insertions of the bits of the small intestine into the sides of the axolotl, between the bases of its legs, have given only primitive formations, similar to those which were obtained after the insertions of cartilages into the sides. 20 homoplastic insertions of the bits of the small intestine walls under the skin of the tail of an axolotl have shown that at the places of the insertions the

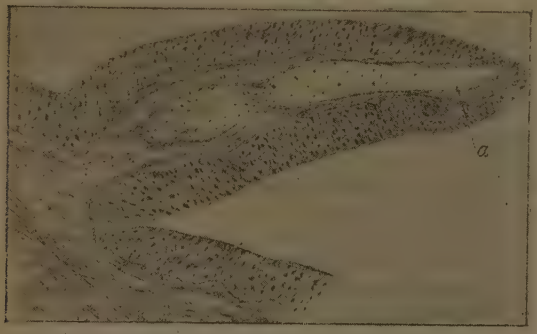

Fig. 2. A longitudinal cut through one of the digits of the didactyl formation *a* shown on Fig. 1.

Fig. 3. The left foreleg of a white axolotl with a tetradactyl additional formation *a* on the summit of the forearm which evolved from under the edge of the wound after the insertion of a bit of the mechanically destroyed tissue of the small intestine.

cutis can be destroyed, and that in this case on the surface of the tail there appear new formations in the shape of semi-transparent tubercles or tonguelets (Fig. 4,*a*). The bits inserted into the regions of the fins have given considerably longer cylindrical formations about 1.5 cm long and bent backwards (Fig. 4, *b*). All of them were situated on the protuberances in the shape of tumours and they consisted inside of a friable connective tissue. In one case we observed the formation of an irregularly shaped cartilage inside the tumour. All these formations were reduced with time. It is necessary to point out that they all greatly resembled the additional formations resulting from the insertion of bits of lung or liver under the skin of the tail; they also had a partial resemblance with certain primitive formations which were obtained in the case of the homoplastic insertion under the skin of the tail of an axolotl of the cartilage of its leg, though they never assumed the form of those tail-like appendages (Fig. 5) which are observed in the highly developed additional formations, appearing in the case of such insertions. The decomposition of the cutis on the tail and the development on the tail of the additional formations after the insertion of the bits of small intestine show that in this case in the tissues there is manifested the activity of the coriocide which, as mentioned above, is probably combined in the small intestine and in the new medium under the skin of the leg with other substances, while its activity disappears. Being placed under the skin of the tail the coriocide is surrounded by a different medium which probably contains a substance breaking the aforesaid con-

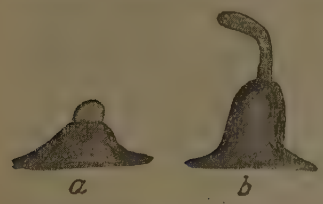

Fig. 4. The formations obtained by the insertion of the bits of the intestine in the shape of a tubercle *a* and of a cylindric appendage *b* bent backwards.

nection*. Being liberated, the coriocide begins its destructive activity. Probably this makes possible the manifestation of the regenerative processes** and, afterwards, the appearance of the additional formation in the case of the insertion of the tissues of the small intestine which contain, as we suppose, various organizing substances. It is necessary to point out that the insertion of 20 bits of the boiled tissue of the small intestine at the edge of the wound failed to produce any new formation; these bits, once being boiled, are probably loosing their ability to destroy the cutis, as well as to develop new formations.

Fig. 5. A tail-like additional formation obtained by the insertion of the cartilage of the leg of an axolotl under the skin of its tail (the length equals 1 cm).

With respect to the heteroplastic insertions, we have made, first of all, 47 insertions of the bits of the small intestine of the crucian under the skin of the legs of an axolotl. In these cases tumours have appeared at the places of insertions, and only in four cases at the edge of the wound there were obtained primitive formations, reaching a height up to 0.1 cm, having the form of buds and similar to those which appeared after the insertion of the bits of the small intestine of an axolotl. These formations were reduced.

In the case of 40 insertions of the bits of the small intestine of a triton (*T. cristatus*) into the legs of the axolotl a greater effect was obtained, namely, besides two primitive formations which were reduced after a lapse of time, there were formed three digit-like formations (115 days after the insertion two of them looked like small tubercles of about 0.5 mm in size), as well as two appendages with two digits containing each a cartilage. One of these appendages was taken for incisions. In the remaining appendage one digit developed, while the other became reduced. In the case of 38 insertions of the small intestine of a frog under the skin of the legs of the axolotl there appeared one digit-like appendage with cartilages undivided into phalanxes inside of it, these cartilages having up to 2 cm in length. 24 subsequent similar insertions produced one digit-like and one didactyl formations with cartilages undivided into phalanxes inside them.

40 insertions under the skin of the leg of an axolotl of the bits of the small intestine of a lizard (*L. vivipara*), as well as 60 similar insertions of the small intestine of a white rat produced no formation. Thus, heteroplastic insertions of the bits of the small intestine are very similar in their results to the corresponding insertions of lung tissues.

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* Such processes are called «activation» by Waddington, Needham and Brochet.

** In my article «The Rôle of Cartilage in the Formations in the Axolotls» I have shown that the processes at the appearance of the additional formations and the processes at the regeneration are of one and the same kind. Therefore, when describing both processes I call them regenerative processes.

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III. GILLS AS ORGANIZER

The new formations which appear after the insertion of the bits of gills under the skin of an axolotl are extremely interesting, mainly in order to find out the kind of action of the organizing substances on the given complex of the tissues of the host. The space available does not permit to give a circumstancial account of the results of study of all the groups of insertions of the bits of gills under the skin of adult axolotls. Therefore, we are limiting our task and we are describing hereunder only the homoplastic insertions, for their results are chiefly deserving our attention.

The gills consist of the following main components: the skin, two longitudinal bunches of muscular fibres, a compact connective tie running along the gills, and blood vessels. All these components, except the latter, were educed and inserted separately under the skin, in order to find out those among them which are containing the organizing substances. For this purpose we have made 25 insertions of the bits of each of these components into the tail. The obtained new formations were primitive and non-differentiated; they had the shape of semi-transparent tubercles and consisted of the epithelium and of the friable connective tissue. From these experiments we can draw the only conclusion that in the development of these new formations there participates the connective tissue, because the skin and the longitudinal tie, when being inserted separately, have produced these formations. It was possible to obtain more complex formations in the case of the insertion, under certain conditions, of all tissues of the gills. For this purpose the gills destined for the insertion were cut into transverse plates. 40 of such bits of the gills were inserted under the skin of the leg. On the surface of the legs there were obtained very feably developed additional formations in the form of small bud-like or coniform protuberances, similar to those additional formations which I have described already on several occasions under the name of the primitive digit-like formations, and which do not contain cartilages inside. The length of each of the 8 formations which I was able to obtain varied from 0.5 mm to 1 mm. Sometimes they had the form of tubercles approaching one another. In these cases the following phenomenon is also observed: on the places of the insertions at the bases of such small protuberances and even, though more rarily, when these protuberances are absent, there appears a depression which opens outwards as a round orifice or as a narrow slit (Fig. 1, *d*). Sometimes, if the depression is small, it becomes wider and has

then a circular cavity. But sometimes, if the depression becomes longer, its walls join one another and they are separated only in some places forming small cavities (Fig. 1, *a*). The depression consists of the epithelial cells with a small number of the Leydig's cells which are usually of a smaller size than those in the skin. There is no cutis on the whole surface of the depression at the beginning of the development. The depression adhers usually to the skin, being pressed to the latter by the neighbouring organs. The depression is up to 2.5 mm long and terminates blindly. Side by side with the above depression there is usually observed a thickening of the epithelium directed inwards or a small depression (Fig. 1, *a*).

It is probable that not all of the substances stimulating the proliferation of the epithelial cells are acting identically; among them there are substances which are stimulating, in various directions, the various parts of the epithelium, and, consequently, there appear the primitive formations in the shape of the external protuberances, as well as of depressions which are forming cavities and tubes similar to those which we observe during the embryonic development.

Sometimes, after the insertion of the organizer, instead of new formations there appear, the cutis being destroyed, the cancer-like irregular expansions of the epithelial cells, as we see this in the case of the insertion under the skin of an axolotl of the cartilage of lizard, while in the case of the insertion under the same skin of the cartilage of an axolotl there appear new formations in the shape of protuberances ([3]). This can be explained by the fact that the substances which are regulating the proliferation of the cells remain inactive in the cartilage of the lizard, i. e. they are combined in it with other substances. Further, the protuberances and the depressons were observed fairly often after the insertion under the skin of the leg of substances contained in some extracts from the cartilage (Fig. 2, *a*). These substances were obtained by dissolving these extracts in water*, alcohol and ether successively. After the evaporation of these extracts in a vacuum, the syrup-like residue was introduced with a microspoon under the skin of the leg of an axolotl. Sometimes we introduced there a bit of the boiled white of a hen egg imbued with the extract. In about 10—20 days there appeared additional formations in the shape of depressions or protuberances of the epithelium.

On the basis of the above, I think, we must admit that after the insertion of a bit of gills into the leg among the stimulators of the proliferation of the epithelium the regulators of this proliferation are manifesting their activity. Probably, these regulators are substances of a twofold kind which are contained in the gill tissue and are combined with other substances, entering into the composition of the gill tissues. After their insertion under the skin the connection is broken, probably owing to the influence of the surrounding medium, and they become active**.

The fact that these regulators are of a twofold kind is confirmed by the results of the subcutaneous insertions of substances which were obtained, by means of extraction, from the cartilage. As we have already seen, they produce

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* In view of the fact that during the extraction by water the proteins contained in the cartilage were dissolved in water, probably a part of the substances which are dissoluble in ether was absorbed by the white and introduced together with it under the skin. The study of the action of the solutions of cartilage are being continued. Here I have only intended to show that the described new formations can appear under the influence of the substances contained in the organizer.

** The suggestion of Needham and my own concerning the means of activation of various substances combined with other substances and consequently exercising their activity are exposed in my article «The Effects ot the Subcutaneous Insertion of Epithelial, Osseous and Muscular Tissues on the Surrounding Tissues in Axolotls»([2]).

the depressions of the external protuberances of the epithelium, while, after the insertion of an entire cartilage under the skin, there is observed no action of the substance producing depressions, but there is only observed the direction of the proliferation of the epithelium for the formation of outward protuberances ([1–3]). Most probably, besides the substance which determines the

Fig. 1. Section through the depression of the epithelium on the leg after the insertion under its skin of a bit of gills; *a*—depression; *b*—small depression; *c*—cutis; *d*—a slit-like external orifice of the depression; *m*—muscular fibres.

above direction of the proliferation of the epithelium and which, for shortness' sake, we will call evaginator, there exists one more substance which is producing the depression. We will call this substance invaginator. In the cartilage this substance is probably combined with one of the substances entering into the composition of the cartilage and it cannot be made active by

Fig. 2. Section through the depression of the epithelium on the leg after the insertion under its skin of the alcohol solution of the cartilage: *a*—orifice of the depression. Other letters indicate the same as on Fig. 1.

the substances under the skin in the medium of the host surrounding the cartilage, while after the insertion of gills this substance becomes liberated. Thus, the organizer plays an important rôle in the appearance of the primitive formations (protuberances and depressions), though at the same time a considerable part is also played by the chemical composition of the medium of the host, for it can either produce the activity of the organizing substance or leave the latter combined in the organizer with other substances which enter into the composition of the organizer.

Of great interest are the formations obtained by the insertion of the bits iof glls under the skin of the tail. We have made two series consisting each of 28 insertions. We have obtained 14 new formations, among which 10 better developed and differentiated formations had the form of appendages which were sometimes narrowed at the base and were provided, on the summit and on the sides, with more or less clearly expressed lobes. In two cases the lobes did not develop and the appendages had the form of a club (Fig. 3, *a*). Their height reached 2 mm. Three formations, somewhat smaller in size, had weakly expressed rudimentary lobes and a very narrowed base (Fig. 3, *b* and *c*). After a certain time they all have fallen off. Finally, three formations, which had the most complex form and reached the size of up to 2.5 mm, looked like wide outgrowths with 3—4 developed lobes (Fig. 3, *d* and *e*) and resembled, in their form, the gills at the beginning of the regeneration of the latter when the petals are appearing on them. One of these new formations began to be reduced, and in this way there was created a two-lobed formation (Fig. *3*, *f*), which was taken for the sections. At any rate, the formations had no resemblance neither with the tail, as we observed after the insertions into the tail of cartilages of the leg, nor with the formations obtained by the insertion of the lung and of the small intestine. Among the minimally developed formations obtained by the insertions of the bits of gills into the tail we found primitive protuberances outwards in the shape of tubercles at the bases of which we were able to observe sometimes the depressions inwards, similar to those obtained by the insertion of the bits of the gills into the leg. In all these phenomena we see an indication that the substances stimulating the proliferation of the epithelium of the host can influence quite considerably the creation of new formations in some form or another; in this respect, there is not always decisive only that part of the body of the host on which the new formation has appeared, as I believed previously when obtaining tail-like appendages after the insertion of cartilages under the skin of the tail ([3]).

The sections have shown that these formations consist of a connective tissue in which, along some of the most developed gill-like formations, there are running bunches of fibres. In one of these formations it was possible to see a small oval region of isolated tissue consisting of cells united by a hyaline substance. It was impossible to investigate thouroughly this tissue, owing to the lack of sufficient data.

Thus, we see that on the tail there can appear new formations of various forms which are determined by the particular organizer which is inserted, i. e., as we suppose, by those of its particular organizing substances which, under the influence of the new medium, are manifesting their activity. We must point out that the more differentiated additional formations appear probably in the case of a more advanced decomposition of the inserted organizer. Thus, I have succeeded in obtaining such more differentiated formations by inserting under the skin of the leg the products of decomposition by the alkaline hydrolysis of the cartilage of the leg. After 30 insertions I have obtained, besides the primitive formations, two additional appendages in the shape of digits and two additional appendages in the shape of an appendix with two digits (Fig. 4, *a*), 5 mm long, which contained cartilages inside. The investigations have not been yet achieved and are still being carried on in this direction.

Besides this, 31 insertions of the bits of gills were made into the back and 34 into the side of the axolotls between the bases of the legs. We have obtained only two formations on the back which had the shape of small outgrowths with tubercles on their summit (Fig. 3, *c*). These formations have developed at the edge of the wound through which the insertion has been made. Far bet-

ter results were obtained by the insertions of 20 bits of gills under the skin of the upper surface of the head. In this latter case we have obtained six formations which had the same shape as the formations on the tail, one formation which was similar to the latter, but differing by two comparatively thiner and longer outgrowths, and two formations which had the shape of an elongated outgrowth narrowed on the summit and provided with small lobes on the sides. One formation was 3.5 mm long and, in its external appearance, it resembled, more than the others, the gills. Another formation was 2.5 mm long; it was considerably curved and it had three lobes on each side. All formations on the head have developed on the wound at its edge.

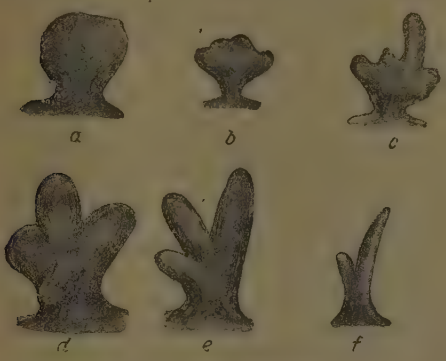

Fig. 3. The external appearance of new formations on the tail after the insertion under its skin of the bits of gills.

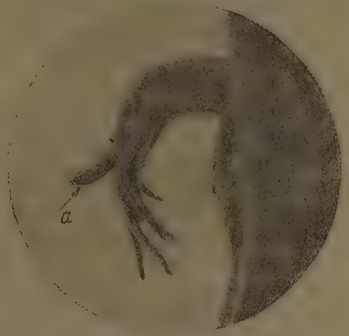

Fig. 4. Additional formation *a* in the shape of a didactyl appendage obtained 90 days after a subcutaneous insertion of the products of the alkaline hydrolysis of the cartilage.

Taking into consideration the results of various insertions* under the skin of adult axolotls aiming to find out the conditions of the development in them of the so-called additional formations**, which results were previously published in several short reports ([1–4]), we arrive at the following conclusions.

1) In the tissues of various organs of vertebrates there can be present substances which are apparently combined with other substances and which we have called organizing substances. The tissues containing them, when introduced under the skin of an adult axolotl, can cause, under certain conditions, certain changes in the tissues of the host, these changes producing the development of the additional formations. Such tissues or their complexes which are contained in certain organs play the rôle of organizers.

2) The dead tissues, as well as some dried ones, do not produce any new formation after their insertion under the skin. The cause of this has not been yet found out.

3) When the inserted tissue remained unchanged and grew fast to the surrounding tissues of the host, no formations whatever have developed.

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* I use the term «insertion» instead of «implantation» because not only tissues, but also various chemical substances were inserted under the skin.

** We have given the name of additional formations to formations which are obtained due to the action of an organizer inserted under the skin and which have a definite form and structure, usually similar to some organs or to their parts or also to the stages of the development of the latter.

4) A preliminary mechanical destruction of some tissues inserted under the skin promotes the decomposition of the cutis, which consists of the connective tissue with collagene fibres, and sometimes it promotes the appearance of more differentiated additional formations.

5) The development of new formations requires a more or less pronounced change of the chemical combination of the inserted tissues of the organizer.

6) In the development of new formations there are observed the necrotic phenomena and the histolysis of the organizer's tissues inserted under the skin.

7) Additional formations, as well as new formations of every kind, which were observed by us, cannot appear unless the cutis is destroyed. The cutis prevents the proliferation of the cells of the epithelium and, thus, does not give the possibility of the appearance of any new formation. The chemical composition of the cutis is probably different in various parts of the body.

8) At the beginning of the development of the additional formations after the insertion of the organizer under the skin of an axolotl there is observed, just at the beginning of the regenerative processes, a stimulation of the proliferation of the epithelial cells and of the connective tissue, as well as there is observed, when the muscular fibres are present, a dedifferentiation and a stimulation of the proliferation of the muscular cells.

9) In the course of further development of the additional formations, as a consequence of the proliferation of the cells of the epithelium in the various layers of the latter, there appear outward the protuberances of the epithelium which are forming bud-like tubercles, or the inward depressions. So there appear formations, which we call primitive and which, to a certain extent, are similar to those which are also observed during the embryonic development. Into the interior of the formed protuberances of the epithelium can enter the mioblasts and the cells of the connective tissue of an embryonic character perhaps due to the mutual cytotaxic attraction between them and the proliferating epithelial cells. At the same time, from the connective tissue or from the perichondrium of the host, there can develop the cartilage. The next stage of the development of the additional formation arrives with the differentiation of the latter.

10) Thus, for the creation of the additional formation the histogenetic material with all its pecularities is provided by tissues of the host surrounding the organizer. These tissues, however, can change under the influence of the organizing substances of the organizer, similarly to the change which occurs in the connective tissue of the host under the influence of the substance which is present in the aqueous extract from cartilage, in the case of a culture of this tissue outside the organism, or in the case of the insertion of lung tissues under the skin.

11) The stimulation of the proliferation of the cells for the creation of new formations is going on probably under the influence of special substances—of stimulators which are present in the organizer inserted under the skin. This is indicated by the cases in which the organizer contains a substance promoting the decomposition of the cutis, as, for instance, in the case of the insertion of cartilage, and when there is observed a direct influence of the organizer on the epithelium. Among the stimulators of the propagation of the epithelial cells in the case of the development of the additional formations there are present apparently substances which are regulating this proliferation and these substances are of a twofold kind: some of them are probably stimulating more intensively chiefly the external layers of the epithelium, thus producing the outward protuberances, while the other stimulate more intensively chiefly the inner layers, producing thus inward depressions.

The existence of these two regulators is evidenced by the appearance, in some of the cases, of a primitive formation of one kind only, namely, of a protuberance, while the depression fails to develop, or vice versa. Probably in the first case there acts one regulator which we have called for shortness' sake evaginator, and in the second case there acts the another regulator, called invaginator. The latter does not manifest its activity when combined with one of the substances of the medium in which it is present. Sometimes both regulators seem to be inactive owing to the same reason. So, after the insertion of the cartilage of a lizard under the skin of the leg of an axolotl there appears an irregular cancer-like proliferation of the epithelium and its growth, both inwardly and outwardly, while the insertion of the cartilage of an axolotl under the same skin produces an additional formation. In general, the cancer-like formations can appear only then, when the cutis is destroyed, as in the given case, and when consequently there exist no obstacles for the development of such formations.

12) The last two items show that for the appearance of the primitive formations there takes place an interaction between the organizer and the place of the host's body into which it was inserted. During the further development of the additional formation, namely, during the differentiation and the development of its form, as well as partly of its structure, there seems also to be indispensable an interaction between the chemical composition of the material of the place of the host's body, into which the organizer is placed, on the one hand, and the composition of the organizer, on the other hand. The former can activate some or other organizing substances of the organizer, i. e. to act in such a way that these organizing substances are liberated from the combination with other substances entering into its composition, and are becoming active. The organizer can exercise a more or less strong influence on the further development of the form and, partly, of the structure of the additional formation and make the latter more or less similar to the organ from which the organizer was taken. If such activization is lacking, the influence of the organizer can become very limited and then the new formation resembles more that part of the host's body on which it has developed*. Such interactions can be very complex, as it can be expected in the case of such complexly organized animals as vertebrates. For the final solution of this problem there are necessary biochemical investigations. There are also necessary the investigations of new formations in adult animals of a simpler organization.

13) The additional primitive formations in the shape of buds or of depressions can be obtained on the legs of an axolotl by means of a subcutaneous insertion of the substances extracted from the cartilage. Besides this, in the case of subcutaneous insertion of the products of the alkaline hydrolysis of the cartilage, i. e. of the product of its more or less full decomposition, there is possible to obtain a more differentiated additional formation. Thus, we see that the substances which are stimulating the propagation of the cells, as well as the regulators which are giving a definite form to new formations, are present in the extracts of their cartilages and among the substances of the products of their hydrolysis. Further investigations along this line are being carried on.

* In the case of interaction between the chemical composition of the organizer, on the one hand, and the place into which it is inserted, on the other, there can also appear formations of an indefinite shape, as it is the case after the insertion of the bits of liver and of intestine under the skin of the tail, as well as mochimeric formations.

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